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Unequal Climate Policy in an Unequal World[1]

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Abstract

We characterize optimal climate policy in an economy with heterogeneous households and non-homothetic preferences. We focus on constrained efficiency, where the planner is restricted from transferring resources across households. We derive three results. First, the constrained-optimal carbon tax is heterogeneous and progressive. Second, if restricted to a uniform tax, the optimal rate is lower than the standard Pigouvian level due to inequality. Third, this allocation is decentralizable using only uniform instruments—a carbon tax, clean subsidy, and a lumpsum transfer. In a quantitative application, we show this policy generates a Pareto improvement, reconciling climate efficiency with inequality concerns.

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1 Introduction

Climate change presents a fundamental challenge for policy design. While carbon taxes are widely viewed as the most efficient tool for reducing emissions, they fall disproportionately on low-income households, whose consumption baskets are significantly more carbon-intensive. In principle, these distributional concerns could be addressed through lump-sum transfers, allowing the tax to focus solely on internalizing the climate externality. In practice, however, the use of such redistributive tools may be restricted, for example, by administrative or political frictions. In this constrained environment, optimal policy must explicitly account for the unequal burden placed on households. We characterize this constrained-efficient policy analytically and quantitatively, showing how the optimal carbon tax is shaped by the interaction between inequality and limits on redistribution.

We build a simple model with clean and dirty goods and households that are heterogeneous by income. Consumption of dirty goods adds carbon to the atmosphere, generating a negative climate externality. When the social planner is allowed to transfer resources across households, the optimal carbon tax is uniform and identical to that from a representative agent economy when the social welfare function is utilitarian. When the planner cannot make transfers between households, the constrained-efficient carbon tax is heterogeneous. If uniformity of the carbon tax is desired, this property must be imposed as an additional constraint. In this case, we show when and how the tax deviates from the optimal carbon tax formula. Importantly, in quantitative exercises we find that this tax policy leads to a Pareto improvement, relative to no policy action.

In the first part of the paper, we combine detailed household expenditure and emissions data to document that the embodied carbon content of household expenditures decreases in income and in wealth. These facts are robust to controlling for household age, education, and family size. These differences can mainly be attributed to low-income households' greater expenditure share on utilities, transportation, and groceries. Motivated by these findings, we incorporate non-homothetic preferences into our model and discipline its calibration by targeting the differences of emission intensities in consumption across households.

To study optimal climate policies in a world with inequality, we build an endowment economy with permanent inequality, where households differ in their labor endowments and supply labor inelastically. Clean and dirty consumption goods are produced using labor. Because the only dynamic variable is the stock of carbon, the household problem is effectively static. This lends tractability to the model and allows us to characterize policies in closed-form. When the planner is not restricted in its instruments, the optimal climate policy is a uniform carbon tax that is equal to the social cost of carbon. This cost is the price of the externality and equals the marginal social damage of carbon divided by the social value of consumption. This means that the planner uses the social value of consumption—the Pareto-weighted average of marginal utilities of consumption—to price the climate externality. In the benchmark case, a utilitarian planner can eliminate inequality by transferring resources across households. This produces the same social value of consumption as that from a representative agent economy, and as a result, the optimal carbon tax is the same in the two economies.

Because our primary objective is to study optimal climate policies in the presence of inequality and not as a means to address inequality, our main results focus on cases that preserve initial inequality. One way to achieve this is by assuming Negishi Pareto weights, where high-income households receive relatively more weight in social welfare. Because these households have lower marginal utilities of consumption, the social value of consumption is lower, resulting in a more aggressive climate mitigation under Negishi. Our central findings come from an alternative approach, where we adhere to a utilitarian planner but impose constrained efficiency (Davila et al., 2012) by restricting its ability to transfer resources across households. As a result, the constrained-efficient carbon tax is heterogeneous. To price the climate externality, the constrained planner uses each household's private value of consumption (i.e., its marginal utility), instead of the social value of consumption. Because the marginal utility of consumption is decreasing with income, the constrained-efficient carbon tax is increasing in income. That is, the constrained-efficient carbon tax is progressive.

Next, we study the optimality of a uniform carbon tax, the policy most commonly proposed in public debates. Since a uniform carbon tax is not the solution to an optimal policy design problem in which the planner is constrained from transferring resources across households, we must impose uniformity of the tax rate as an additional constraint. Even though this planner cannot do away with inequality, the uniform constrained-efficient carbon tax nevertheless follows the same formula as that of the unconstrained planner. That is, the uniform constrained-efficient carbon tax is set to the marginal social damage of carbon divided by the social value of consumption. However, because marginal utilities are heterogeneous in this case, the social value of consumption can differ from that of the unconstrained planner.

When the marginal utility function is sufficiently convex, climate mitigation under a uniform constrained planner is moderated relative to the utilitarian optimal benchmark. The intuition for this result is simple. Because the weighted average of marginal utilities is greater than the marginal utility of average consumption, the social value of consumption is higher for the constrained planner. Unable to eliminate initial inequality and prohibited from assigning heterogeneous tax rates, the planner accounts for the high marginal utilities of low-income households by effectively putting more weight on the social value of consumption relative to the social damage from carbon emissions.

With further functional form assumptions on preferences, we can provide an alternative implementation of the uniform constrained-efficient allocation that does not include individual rebates. Specifically, the uniform constrained-efficient allocation can be decentralized through a combination of three instruments: a uniform carbon tax, a uniform clean subsidy, and a uniform lump-sum transfer. This implementation is appealing as it places a lower informational requirement on policymakers.

We assess the welfare and climate implications of these carbon taxes through a numerical exercise. Among climate policies that maintain initial inequality, we find that the heterogeneous constrained-efficient carbon tax achieves the highest average welfare gain and the most significant long-term temperature reduction. However, due to its progressive structure, the constrained-efficient tax faces limited support among high-income households, which bear a greater tax burden and experience welfare losses. In contrast, the uniform constrained-efficient tax secures full support (i.e., it is Pareto-improving) and achieves the highest average welfare gain from among the uniform taxes. Nevertheless, universal support comes at the expense of the climate. The long-term global temperature reduction is the smallest under this tax policy. Greater temperature reduction occurs under the uniform Negishi tax, but this policy faces substantial opposition, now from the bottom of the distribution.

Finally, we embed the simple model in a standard incomplete markets model with idiosyncratic labor income risk. This empirically grounded economy captures existing inequality and fiscal policy, enabling us to quantify both the effectiveness and the distributional consequences of carbon taxes. The economic parameters are calibrated to match key features of the US income and wealth distributions as well as US fiscal policy. Climate parameters are calibrated to ensure that economic activity generates global emissions consistent with the data and contributes to temperature rises that are consistent with recent estimates.

We solve the model under several climate policy scenarios. First, we use the uniform constrained-efficient tax formulas to solve for the uniform carbon tax combined with a household-specific transfer that fully rebates each household's carbon tax payment. The equilibrium carbon tax schedule begins at \$48 per ton and rises gradually over time to a long-run value of \$89 per ton, reflecting the increasing social cost of carbon. Next, we solve for a proxy to the heterogeneous constrained-efficient carbon tax where each household's carbon tax rate is a function of its current labor productivity. The value of the initial carbon tax ranges from \$17 per ton for the poorest households to nearly \$2,500 per ton for the richest. Just as in the uniform case, all tax rates rise over time as the social cost of carbon increases.

Both policies reduce carbon emissions substantially and lower temperatures relative to business-as-usual (BAU), but the greatest climate gains emerge far in the future. The heterogeneous carbon tax, which induces the strongest reduction in carbon accumulation, lowers long-run temperatures by approximately 1.1 degrees Celsius relative to BAU. These carbon tax policies are welfare improving for the majority of households. The uniform carbon tax, in particular, achieves a Pareto improvement relative to BAU. Welfare gains in the initial period of transition are small on average (about 0.05 percent in consumption equivalence units); however, they grow substantially over time due to accumulated climate improvements relative to BAU.

Comparing the uniform and heterogeneous tax cases, the latter produces a somewhat lower temperature path and a slightly higher average welfare gain. In both cases, the welfare benefit of climate mitigation is greatest among the wealthy, who place a greater value on the environment due to their lower marginal utility of consumption. Consequently, under a uniform carbon tax, welfare gains increase monotonically in both wealth and income (i.e., labor productivity). In contrast, by taxing high-income households more, the heterogeneous carbon tax generates welfare gains that increase in wealth but decrease in income.

We consider several extensions. First, we connect our results to the literature on alternative uses of carbon-tax revenues. In contrast to the constrained-efficient policies we study, these approaches allow for explicit redistribution across households. Consistent with existing findings, we show that lump-sum recycling delivers sizable welfare gains by mitigating the regressive effects of the tax and providing insurance. Second, we examine in the Appendix the implementation of the constrained-efficient allocation in an economy with incomplete markets. Implementation requires optimal income taxation to correct underlying market inefficiencies (Davila et al. 2012), in addition to the carbon tax. We show that the constrained-efficient carbon-tax formulas derived in the main text remain optimal in this environment.

Finally, we show that our main quantitative result – that a uniform carbon tax with rebates delivers a Pareto improvement – is robust to alternative modeling assumptions and parameter values. In particular, we extend the model to allow for heterogeneous climate damages, with damages that are larger for low-income households. For empirically plausible values of the elasticity of damages with respect to income, uniform carbon taxes with rebates continue to deliver a Pareto improvement. Higher risk aversion increases the convexity of marginal utility, raising the social value of consumption and resulting in a lower optimal carbon tax and more moderate mitigation. Greater impatience lowers the social cost of carbon by placing less weight on future damages. In both cases, however, the uniform carbon tax with rebates remains Pareto improving.

This paper contributes to a growing body of literature that builds on the foundational work of Nordhaus and Boyer (2003) (and its contemporary version, Golosov et al. 2014) on representative agent neoclassical growth economies with climate dynamics by incorporating heterogeneous agent economies with incomplete markets. Recent notable contributions come from Hillebrand and Hillebrand (2019), Krusell and Smith Jr. (2022), Cruz and Rossi-Hansberg (2024), and Conte et al. (2025) who analyze climate policy within a spatial economic framework, and Fried et al. (2018, 2024), Douenne et al. (2024, 2025), Wöhrmüller (2024), and Kubler (2025) who focus on household heterogeneity and fiscal policy.

Our analysis engages with an expanding literature on optimal climate policy in the presence of fiscal distortions and heterogeneity. Seminal work by Barrage (2020) differentiates between utility and production damages and studies how the optimal carbon tax in a representative agent setting interacts with distortionary fiscal policy. Our paper is most closely related to a recent strand of the literature that considers fiscal reforms to address climate change in economies with heterogeneous agents. Fried et al. (2024) build a life-cycle model where households have non-homothetic preferences for dirty consumption and study the welfare consequences of alternative ways to rebate revenue from a given carbon tax. In contrast, we theoretically characterize the constrained-efficient climate policy (taxes and transfers) in the economy with heterogeneity and use these characterizations to inform the policies studied in our quantitative exercises. Jacobs and van der Ploeg (2019), Douenne et al. (2024) and Douenne et al. (2025) study optimal policy to resolve climate change and inequality concerns when carbon taxes are considered jointly with other distortionary fiscal instruments. We differ from these papers in that we design carbon taxes and transfers to fix a climate externality in the presence of inequality rather than to address climate and inequality simultaneously.

We do so by restricting attention to constrained-efficient allocations as in Davila et al. (2012), where resource transfers across agents are ruled out. Bourany (2024) also studies constrained-efficient allocations within climate-economy models with idiosyncratic risk and incomplete markets. Our paper differs in scope in that we focus on carbon taxation in the presence of consumption inequality across households, rather than regional heterogeneity across countries. Like these papers, we emphasize the inherent tension between efficiency and redistributive objectives that arises in policy design in heterogeneous-agent economies.

This tension between efficiency and equity in the climate problem has long been recognized in the literature, including Azar and Sterner (1996), Fankhauser et al. (1997), Anthoff et al. (2009), Nordhaus (2011), Anthoff and Emmerling (2019), Anthoff and Tol (2010), Dennig et al. (2015), and Nordhaus and Yang (1996). This body of work studies alternative Pareto weighting schemes, often emphasizing Negishi weights, and highlights the role of the intertemporal elasticity of substitution as a key parameter governing inequality aversion. We contribute to this literature by comparing the optimal allocation under a Negishi planner with the constrained-efficient allocation that arises when inequality is preserved not through welfare weights, but because resource transfers across households are explicitly ruled out.

The paper also connects to the literature that studies the distributional role of carbon tax revenue, such as Rausch et al. (2011), Pizer and Sexton (2019), Fullerton and Monti (2013), and Goulder et al. (2019), as well as the literature on the "double dividend" hypothesis. While standard theory suggests recycling carbon revenues to lower distortionary taxes, our results are consistent with the literature on incomplete markets suggesting that this logic does not necessarily hold in the presence of uninsurable risk as the demand for insurance through redistribution dominates efficiency concerns in this class of models (Fried et al. 2018, Dyrda and Pedroni 2023, Carroll and Hur 2023, and Carroll et al. 2024). Our empirical work is related to Grainger and Kolstad (2010) and Sager (2019), who document how emissions embodied in household expenditures vary with income. We extend this literature by providing a more detailed mapping of expenditures to emissions and by documenting that the embodied carbon content of consumption decreases not only in income, but also in wealth. Consistent with our empirical finding, Känzig (2025) documents that carbon prices affect the consumption of low-income households relatively more. However, our paper differs in scope and methodology, as we set up an optimal policy design problem to examine carbon pricing policies.

The remainder of this paper is organized as follows. Section 2 uses data on household expenditure and embodied emissions to document how emission intensities differ across income and wealth. Section 3 presents a simple model with unequal agents and climate change, and Section 4 provides the main theoretical results. Section 5 presents the quantitative model, the calibration, and the quantitative results. Finally, Section 6 concludes.

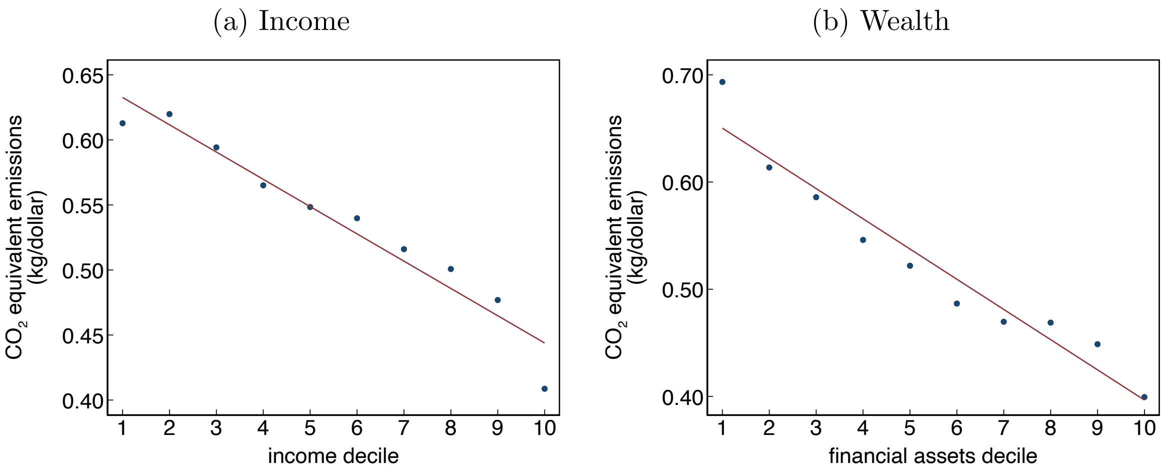
2 Data

In this section, we document the embodied emissions content of household expenditures and how it differs across the income and wealth distribution. We combine household expenditure data from the Consumer Expenditure Survey (CEX) with industry-level emissions data from the Environmental Protection Agency (EPA). Unlike prior studies that often rely on coarser sector-level aggregations, we construct a granular concordance mapping 671 Universal Classification Codes (UCC) from the CEX to 394 North American Industry Classification System (NAICS) codes used in the EPA dataset. This detailed mapping allows us to capture heterogeneity in carbon intensities often lost in broader aggregation. We provide the full details of this data construction in Appendix A.

Consistent with Grainger and Kolstad (2010) and Sager (2019), we find that emission intensities decrease with income. Importantly, we show that this relationship extends strongly to wealth. While the CEX only contains data on liquid wealth, we show that the patterns are similar when using the Panel Study of Income Dynamics (PSID), which contains a more complete representation of household wealth (see Appendix A.2).

Figure 1 plots average embodied emissions per dollar spent by income and liquid wealth decile. There is a clear negative gradient: compared with the highest wealth households, the expenditure of the lowest-wealth households is associated with approximately 50 percent more CO_2-equivalent emissions per dollar spent. Regression analysis confirms that this negative association remains statistically significant at the 1 percent level even after controlling for household characteristics such as age, education, and family size (see Appendix Table A.1).

Figure 1: Embodied emissions


(a) Income

(b) Wealth

To understand the drivers of this intensity gradient, we decompose household expenditures into broad expenditure categories. Table 1 reveals that low-wealth households allocate a significantly larger share of their budget to carbon-intensive categories—specifically utilities, transportation, and groceries—compared to high-wealth households. Conversely, high-wealth households spend relatively more on service-oriented categories, such as education and health care, which have lower carbon intensities.

Table 1: Embodied emissions and expenditure shares

Expenditure category	Embodied emissions (CO_2 kg/dollar)	Expenditure shares (percent) Low wealth	Expenditure shares (percent) High wealth
Utilities	1.71	10.3	5.0
Transportation	1.16	17.8	17.2
Food and beverages at home	0.80	15.7	7.6
Other expenditures	0.11	56.2	70.2

High and low wealth correspond to the top and bottom deciles of liquid wealth, respectively, conditional on working age.

Based on these findings, in the next sections, we build and characterize a model with non-homothetic preferences that allows for poor households to have higher emission intensities embodied in their consumption.

3 A Simple Model

In the previous section, we documented that the emissions embodied in household expenditures substantially vary with income and wealth, suggesting that a carbon tax can have unequal consequences across households. In this section, we develop a simple model of unequal households and climate change to study how the optimal carbon tax depends on underlying inequality.

Consider an economy populated by a continuum of households, indexed by i with measure μ_i. There are two consumption goods, clean and dirty: c_{ct} and c_{dt}. Consumption of the dirty good adds carbon to the atmosphere, S_t, which evolves according to:

$$S_{t+1} = (1-\delta)S_t + \upsilon \sum_i \mu_i c^i_{dt} \tag{1}$$

where δ is the natural rate of carbon re-absorption and υ is the carbon content of dirty good consumption.

Households' preferences over consumption and carbon are given by:

$$\sum_{t=0}^{\infty} \beta^t \left[u(c_{ct}, c_{dt}) - x(S_{t+1}) \right] \tag{2}$$

where $x(S)$ is the climate damage function with $x'(S) > 0$ and $x''(S) > 0$. The function x subsumes the welfare losses from the presence of carbon in the atmosphere, and we assume these losses take the form of a utility cost. We assume that preferences are additively separable in consumption and carbon damages, following Nordhaus and Boyer (2003) and subsequent work. Thus, atmospheric carbon affects the economy only through its effect on welfare. In the quantitative exercise, we additionally consider the mapping from the carbon stock to the global temperature, and we include in x all climate-related welfare losses regardless of whether they are utility or output related. Therefore, x in the model represents global climate change. Households are endowed with ε_i units of labor (inelastically supplied).

We begin by characterizing the socially optimal allocation as the solution to a planner's problem that weights households' welfare according to a given Pareto weighting scheme:

Definition 1 (Optimal Allocation) *Let $\{\alpha_i\}_{\forall i}$ be an arbitrary set of Pareto weights with $\sum_i \alpha_i \mu_i = 1$. The socially optimal allocation is the sequence $\{c^{i\star}_{dt}(\alpha_i), c^{i\star}_{ct}(\alpha_i), S^{\star}_t\}_{t=0}^{\infty}$ that solves the social planner's problem, which is to maximize*

$$\sum_i \alpha_i \mu_i \left[\sum_{t=0}^{\infty} \beta^t \left(u(c^i_{ct}, c^i_{dt}) - x(S_{t+1}) \right) \right] \tag{3}$$

subject to the carbon cycle (1) and the resource constraint,

$$\sum_i \mu_i c^i_{ct} + \sum_i \mu_i c^i_{dt} = \sum_i \mu_i \varepsilon^i. \tag{4}$$

Because climate damages are additively separable and homogeneous across households, the welfare function (3) can be rewritten as

$$\sum_i \alpha_i \mu_i \sum_{t=0}^{\infty} \beta^t u(c^i_{ct}, c^i_{dt}) - \sum_{t=0}^{\infty} \beta^t x(S_{t+1}) \tag{5}$$

which makes explicit the separability of the climate externality—a feature that lends useful tractability to our analysis. In Appendix E, we extend the model to allow for heterogeneous climate damages and conduct a sensitivity analysis.

The first order conditions for this problem are:

$$(c^i_{dt}) : \alpha_i u^i_{dt} - \upsilon\sigma_t - \lambda_t = 0, \tag{6}$$

$$(c^i_{ct}) : \alpha_i u^i_{ct} - \lambda_t = 0, \tag{7}$$

$$(S_{t+1}) : -x'(S_{t+1}) + \sigma_t - \beta\sigma_{t+1}(1-\delta) = 0, \tag{8}$$

where $\beta^t\sigma_t$ and $\beta^t\lambda_t$ are the Lagrange multipliers on the carbon cycle and resource constraint, respectively. Iterating forward from (8), we have:

$$\sigma_t = \sum_{j=1}^{\infty} [\beta(1-\delta)]^{j-1}\, x'(S_{t+j}), \tag{9}$$

representing the present discounted sum of climate-induced damages associated with an additional unit of carbon, which we will refer to as the *marginal social damage* of carbon.[2]

Notice that equations (6)–(7) hold for all i. Thus, for all i

$$\lambda_t + \upsilon\sigma_t = \alpha_i u^i_{dt}, \tag{10}$$

$$\lambda_t = \alpha_i u^i_{ct}. \tag{11}$$

That is, weighted marginal utilities are equated across agents. This implies that, for all i, j,

$$\frac{u^i_{dt}}{u^i_{ct}} = \frac{u^j_{dt}}{u^j_{ct}}, \tag{12}$$

meaning that the marginal rates of substitution between goods are equated across agents.

Combine (10)–(11) to obtain:

$$1 + \frac{\upsilon\sigma_t}{\lambda_t} = \frac{u^i_{dt}}{u^i_{ct}}. \tag{13}$$

The optimality condition says that the marginal utility must be equal across goods, after taking into account the climate externality.

We begin by discussing some properties of the optimal carbon tax in a setting with unrestricted lump-sum transfers. The tax implements the optimal path of carbon emissions

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[2] When climate damages take the form of an output loss, the marginal social damage includes these losses. Suppose a fraction of the endowment is lost due to climate damages so that net endowment is $[1 - v(S_{t+1})]\varepsilon^i$ with $v(0) = 0$ and v increasing and convex. Then (9) generalizes to:

$$\sigma_t = \sum_{j=1}^{\infty} [\beta(1-\delta)]^{j-1} \left[x'(S_{t+j}) + \lambda_{t+j} v'(S_{t+j})\right]$$

where λ_{t+j} is the social value of consumption at time $t+j$. All analytical results in this section extend to this more general case.

as a competitive equilibrium with taxes, setting a relative price to carbon emissions in units of the consumption good. In an economy with inequality, the value of consumption depends on a weighted average of households' marginal utilities across income groups, a term that will play a central role in the alternative policy scenarios we explore below.

Lemma 1 (Optimal Carbon Tax) *The optimal Pigouvian tax, $\tau_t^\star$, that implements the socially optimal allocation is*

$$\tau_t^\star \equiv \frac{\upsilon \sigma_t}{\sum_i \alpha_i \mu_i u_{ct}^i}. \tag{14}$$

with lump-sum transfers equal to $T_t^i(\alpha_i) = (1+\tau_t^\star)c_{dt}^i + c_{ct}^i - \varepsilon^i$.

The proof is in Appendix D. The optimal Pigouvian tax in (14) reflects the social cost of carbon, which is the ratio of the marginal social damage to the social value of consumption, measured in this case by the Pareto-weighted average marginal utility of clean consumption. An important feature of this tax is its uniformity across households: as in a representative-agent economy, a uniform carbon tax remains optimal in the presence of household heterogeneity, provided that lump-sum transfers are available. Moreover, under a utilitarian planner ($\alpha_i = 1\ \forall i$), equation (14) simplifies to

$$\tau_t^{\mathbf{U}} = \frac{\upsilon \sigma_t}{u_{ct}}. \tag{15}$$

where $\tau_t^{\mathbf{U}}$ denotes the utilitarian optimal carbon tax and u_{ct} is the marginal utility of clean consumption. In this case, the utilitarian planner equalizes marginal utilities across households, eliminating consumption inequality overall. As a result, the optimal allocation coincides with that of a representative-agent economy, and the optimal tax is the same. Importantly, however, both tax schemes $(\tau_t^\star, \tau_t^{\mathbf{U}})$ entail a significant redistribution of resources across households implemented through transfers.

4 The Constrained-Efficient Carbon Tax

With unrestricted lump-sum transfers, the planner can eliminate existing inequality. Since our goal is to study carbon taxes as a tool to correct the climate externality—not to address inequality—it is useful to focus on policies that take the initial income distribution as given. These policies offset the income effects induced by carbon taxation but do not otherwise redistribute resources across households. There are two natural ways to do this. The first, more commonly discussed in the climate literature, is to use Negishi Pareto weights, which place greater weight on high-income households and therefore preserve the initial inequality. The second is to retain a utilitarian welfare function while explicitly ruling out any redistribution across households. We study both approaches below.

4.1 Negishi Optimal Carbon Tax.

Negishi weights are proportional to the inverse of the marginal utilities of individual's total consumption. Using (14), the Negishi carbon tax, $\tau_t^{\mathbf{N}}$, is equal to

$$\tau_t^{\mathbf{N}} = \frac{\upsilon \sigma_t}{\sum_i \frac{\frac{1}{u_\varepsilon^i}\mu_i}{\sum_j \frac{1}{u_\varepsilon^j}\mu_j} u_{ct}^i}. \tag{16}$$

where u_ε^i denotes the marginal utility of consuming the individual endowment. Each household receives a lumpsum rebate of their tax bill so that individual transfers equal $T_t^i = \tau_t^{\mathbf{N}} c_{dt}^i$ for every period t.

Relative to a utilitarian planner, the Negishi planner has less concern for the plight of poor households. Consequently, it can be shown that the Negishi planner assigns a lower social value of consumption and taxes carbon emissions more aggressively. However, the tax is high not because the climate externality is necessarily large, but because the planner assigns little weight to the high marginal utility of poor households.

4.2 Constrained-Efficient Carbon Tax

A Negishi planner gives more weight to richer households and less weight to poorer ones, and reproduces the existing inequality by the choice of the Pareto weights. A utilitarian planner, in contrast, treats all individuals symmetrically and assigns equal weight to each household. This "veil-of-ignorance" perspective is the standard approach in social welfare analysis. Under utilitarianism, inequality matters because marginal utilities differ across income groups. In this section, we therefore keep a utilitarian objective but explicitly rule out any redistribution of resources in a constrained-efficient problem, as in Davila et al. (2012). To do so, we consider a government that rebates the proceeds from carbon taxation back to each household according to its tax bill. This tax and transfer scheme distorts the consumption bundles of households while leaving the underlying distribution of resources unchanged.

In a decentralized environment, there are two production units, the clean and the dirty good producers indexed by j. A representative firm in each sector uses labor as the only input according to a linear technology. Households maximize utility (2) subject to the following set of budget constraints,

$$p_t(1+\tau_t^i)c_{dt}^i + c_{ct}^i \leq w_t\varepsilon^i + T_t^i, \tag{17}$$

for every period t, where p_t is the relative price of dirty to clean consumption, w_t is the wage, and (τ_t^i, T_t^i) are a carbon tax and a lump-sum transfer. With linear technology, market prices satisfy $w_t = p_t = 1$.[3] To rule out redistribution, transfers are equal to

$$T_t^i = \tau_t^i c_{dt}^i \tag{18}$$

for all i and t. As is standard, households take the lumpsum transfer as an exogenous amount. Plugging the transfer scheme (18) into the household budget constraint, the planner is now constrained to consider only allocations that satisfy the following implementability condition:

$$c_{ct}^i + c_{dt}^i \leq \varepsilon^i \tag{19}$$

for all i and t.

Condition (19) is certainly more restrictive than the feasibility condition (4) and prevents the utilitarian planner from pursuing any direct redistribution.

Definition 2 (Constrained-Efficient Allocation) *The constrained-efficient allocation is the sequence* $\{c_{dt}^i, c_{ct}^i, S_t\}_{t=0}^{\infty}$ *that solves the constrained-efficient utilitarian social planner's problem, which is to maximize social welfare (3), with* $\alpha_i = 1$ *for all* i*, subject to the carbon cycle (1) and the implementability condition* (19).

The first-order conditions for this problem are:

$$(c_{dt}^i) : u_{dt}^i - \upsilon\sigma_t - \lambda_t^i = 0 \tag{20}$$

$$(c_{ct}^i) : u_{ct}^i - \lambda_t^i = 0 \tag{21}$$

$$(S_{t+1}) : -x'(S_{t+1}) + \sigma_t - \beta\sigma_{t+1}(1-\delta) = 0 \tag{22}$$

where $\beta^t\mu_i\lambda_t^i$ is the Lagrange multiplier on the implementability condition (19).

Combine equations (20) and (21) to obtain:

$$\frac{u_{dt}^i}{u_{ct}^i} = 1 + \frac{\upsilon\sigma_t}{u_{ct}^i} \tag{23}$$

In contrast to the unconstrained optimal allocation, the marginal utilities in (20) and (21) and the marginal rate of substitution between consumption of clean and dirty goods in (23)

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[3] A detailed description of the competitive equilibrium in the simple economy is in Appendix B.

are not equal across all households. It follows that the constrained-efficient carbon tax is no longer uniform. The following proposition characterizes the constrained-efficient carbon tax. The proof is in Appendix D.

Proposition 1 (Constrained-Efficient Carbon Tax) *Suppose that the constrained-efficient allocation is* $\{c^i_{dt}, c^i_{ct}, S_t\}_{t=0}^{\infty}$ *for all* i*. Then, there exists a sequence of prices* $\{w_t, p_t\}_{t=0}^{\infty}$ *such that the allocation is a competitive equilibrium with taxes given by*

$$\tau^i_t = \frac{\upsilon \sigma_t}{u^i_{ct}} \quad \forall i. \tag{24}$$

The tax revenue is rebated back to each household with transfers equal to $T^i_t = \tau^i_t c^i_{dt}$ *for every period* t *and for all* i*.*

For each household, the constrained-efficient carbon tax in (24) is set to the social cost of carbon, valued at its own marginal utility. That is, even though all households share the same climate damages in absolute terms, they differ in how much they value the climate damages relative to their own consumption. Because households with lower income have a higher marginal utility, it follows that

$$\tau^j_t < \tau^k_t$$

for all j and k with $\varepsilon^j < \varepsilon^k$. Therefore, the constrained-efficient carbon tax calls for a higher rate for households with higher incomes.[4] Notice that, absent any transfer of resources across households, some redistribution of welfare still occurs indirectly by distorting more the consumption bundle of high-income households. Intuitively, the planner assigns a higher carbon tax to the rich because reducing their dirty consumption is less costly in terms of social welfare than reducing the dirty consumption of the poor.

4.3 Constrained-Efficient Uniform Carbon Tax

A uniform carbon tax is common in policy proposals and is appealing because of its simplicity. However, a uniform tax is not optimal in a heterogeneous economy where transfers across households are ruled out. Recovering a uniform carbon tax therefore requires imposing tax uniformity as an additional constraint in the planner's problem. At the allocation level, this

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[4] This result is related to Chichilnisky and Heal (1994), who show in a multi-country setting that Pareto efficiency requires the marginal cost of abatement of a global externality to be inversely related to a country's marginal valuation of consumption. Since richer countries have a lower marginal valuation of consumption, they should optimally bear a higher abatement cost.

constraint implies that the marginal rate of substitution between clean and dirty consumption must be equalized across households:

$$\frac{u^i_{dt}}{u^i_{ct}} = \frac{u^j_{dt}}{u^j_{ct}} \tag{25}$$

for all i, j. Furthermore, for preferences of the form

$$u(c_{ct}, c_{dt}) = \frac{\left((c_{ct} + \bar{c})^{\gamma}\ c_{dt}^{1-\gamma}\right)^{1-\kappa}}{1-\kappa} \tag{26}$$

the constraint can be written as

$$\left(c^i_{ct} + \bar{c}\right) c^j_{dt} = \left(c^j_{ct} + \bar{c}\right) c^i_{dt} \tag{27}$$

for all i, j.[5] The parameter γ represents preference over clean consumption and $\bar{c} > 0$ is the non-homotheticity parameter, which allows the model to match the differences in embodied emission intensities across households documented in Section 2. Additionally, we assume that $\kappa > 1$.

The following proposition characterizes the constrained-efficient climate policy in an economy where the planner is fully constrained from using climate policy to redistribute resources across households. In the model, this restriction implies no direct transfer of resources across individuals and uniform carbon taxes. The proof can be found in Appendix D.

Proposition 2 (Constrained-Efficient Uniform Carbon Tax) *Suppose that the allocation* $\{c^i_{dt}, c^i_{ct}, S_t\}^{\infty}_{t=0,\forall i}$ *solves the constrained-efficient planner's problem with the additional constraint* (27)*. Then, there exists a sequence of prices* $\{w_t, p_t\}^{\infty}_{t=0}$ *such that the allocation is a competitive equilibrium with taxes given by*

$$\tau_t = \frac{\upsilon \sigma_t}{\sum_i \frac{\mu_i c^i_t}{\sum_j \mu_j c^j_t} u^i_{ct}} \tag{28}$$

with $c^i_t \equiv c^i_{ct} + \bar{c} + c^i_{dt}$*. The revenue is rebated back with transfers equal to* $T^i_t = \tau_t c^i_{dt}$ *for every period* t *and for all* i*.*

The constrained-efficient uniform carbon tax follows the Pigouvian rule, by using a weighted average of marginal utilities to price the climate externality. When the marginal

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[5] Though we make this functional form assumption here for exposition purposes, it is not necessary for the proof of Proposition 2.

utility function is sufficiently convex, climate mitigation under a uniform constrained planner is moderated relative to the utilitarian optimal benchmark. The intuition for this result is simple: Because the weighted average of marginal utilities is greater than the marginal utility of average consumption, the social value of consumption is higher for the constrained planner. For a given marginal social damage, this will result in a lower carbon tax relative to both the utilitarian optimal tax and the Negishi optimal tax, provided that the risk aversion κ is greater than one. In fact, when $\kappa = 1$, the formulas for the utilitarian optimal carbon tax, Negishi optimal carbon tax, and uniform constrained-efficient tax are identical, and in particular, the Negishi optimal and uniform constrained-efficient allocation and tax rates are identical.

An Alternative Decentralization. The implementation of the uniform-constrained carbon tax with individual lump-sum rebates can be challenging to administer since the government would have to issue lumpsum transfers that exactly offset the carbon tax revenue associated with each household's consumption. In the next result, we show that the constrained-efficient allocation can be decentralized without household-specific rebates by using a combination of uniform instruments: a uniform carbon tax, a uniform subsidy on clean consumption, and a uniform lump-sum transfer. In particular, consider an alternative market economy where households face a carbon tax on dirty consumption, τ_{dt}, a clean subsidy, τ_{ct}, and lump-sum transfers, T_t. The household's problem is to maximize (2) subject to the following set of budget constraints,

$$p_t(1 + \tau_{dt})c^i_{dt} + (1 - \tau_{ct})c^i_{ct} \leq w_t\varepsilon^i + T_t \tag{29}$$

for every period t. The first-order conditions for this problem lead to the following optimality condition:

$$\frac{u^i_{dt}}{u^i_{ct}} = \frac{1 + \tau_{dt}}{1 - \tau_{ct}} \tag{30}$$

where the marginal rate of substitution between clean and dirty consumption equals the relative after-tax price of the goods. As before, optimality on the firm's side implies that $p_t = w_t = 1$.

Corollary 1 (Uniform Carbon Tax, Clean Subsidy, and Transfer) *Suppose that the allocation* $\{c^i_{dt}, c^i_{ct}, S_t\}^\infty_{t=0,\forall i}$ *solves the constrained-efficient planner's problem with the additional constraint* (27)*. Then, the constrained-efficient allocation* $\{c^i_{dt}, c^i_{ct}, S_t\}^\infty_{t=0,\forall i}$ *is also implementable as a competitive equilibrium with an all-uniform climate policy* $\{\tau_{dt}, \tau_{ct}, T_t\}$

given by:

$$\tau_{dt} = \gamma\tau_t \quad ; \quad \tau_{ct} = (1-\gamma)\frac{\tau_t}{1+\tau_t} \quad ; \quad T_t = \tau_{ct}\bar{c} \tag{31}$$

where τ_t *is given by* (28) *from Proposition 2.*[6]

The proof is in Appendix D. This alternative policy can implement the constrained-efficient allocation, providing a practical alternative to the uniform constrained efficient carbon tax with individual rebates since the government only needs information on the distribution of consumption and an estimate for the social cost of carbon to compute and implement the optimal tax. In the special case of homothetic preferences, identical expenditure shares allow for implementation using only a carbon tax and clean subsidy. This corollary establishes that this decentralization result holds even with non-homothetic preferences—an important finding given the heterogeneity documented in Section 2—thereby eliminating the need for the individual-specific rebates discussed earlier.

4.4 A Numerical Illustration

In this section, we numerically illustrate the welfare and climate consequences of the various optimal tax rules characterized in Section 4, none of which involve net transfers across households. We assume that the labor endowments, ε^i, are distributed according to a log normal distribution, parameterized to match the consumption inequality of the quantitative model introduced in the next section. The remaining parameters are chosen to align with observed moments related to emissions embedded in household expenditures, total global emissions, and estimates of climate damages.

Table 2 reports the welfare and climate consequences of implementing the heterogeneous constrained-efficient tax from Proposition 1, the uniform constrained-efficient carbon tax from Proposition 2, and the Negishi optimal tax from equation (16). The numerical exercise has two main takeaways. First, the heterogeneous constrained-efficient carbon tax delivers the highest average welfare gain and largest long-run temperature reduction, though households in the top decile of the income distribution suffer a welfare loss, due to the very distortionary carbon taxes levied on them.

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[6] We derive this implementation using Stone-Geary preferences to obtain tractable closed-form expressions for the tax and subsidy instruments. However, the result that the constrained-efficient allocation can be implemented with uniform instruments generalizes to the class of preferences admitting the Gorman Polar Form (Gorman 1953; Jacobs and van der Ploeg 2019). This class, which includes quasilinear preferences, implies linear and parallel Engel curves, ensuring that the marginal propensity to consume dirty goods is identical across households.

Table 2: Welfare and Climate

Policy	Initial carbon tax (\$/ton)	Long-run temp. reduction (degrees)	Average welfare gain (percent)	Support (percent)
Constrained-efficient tax	111.7*	1.7	0.067	91.5
Uniform constrained-efficient tax	57.7	1.3	0.060	100.0
Negishi optimal tax	78.74	1.6	0.059	89.1

Note: The model is parameterized as follows: $\log \varepsilon \sim \mathrm{N}(0, 0.31)$, $x(S) = \frac{\Psi}{2} S^2$, $\gamma = 0.98$, $\bar{c} = 0.27$, $\Psi = 0.02$, $\beta = 0.97$, $\kappa = 2$, $\delta = 1/300$, and $\upsilon = 326.4$. * denotes the average carbon tax rate, weighted by dirty consumption expenditures.

Second, if we compare the two uniform carbon taxes, one in which the utilitarian planner is restricted from transferring resources across households (uniform constrained-efficient), and the other in which the planner weights households in such a way that there is no incentive to transfer resources across households (Negishi optimal), the uniform constrained-efficient tax delivers not only slightly higher welfare gains on average, but is also the only policy that is Pareto improving. By putting higher weight on high-income households, which value consumption less than low-income households, the Negishi planner sets a carbon tax that is higher than what most low-income households would prefer, resulting in a welfare loss for the bottom decile of the income distribution. For this and other technical reasons, we henceforth focus on the utilitarian constrained optimal taxes.[7]

In the next section, we conduct a quantitative exploration of the utilitarian constrained-efficient tax rates in an extended quantitative model.

5 Quantitative Analysis

For the quantitative analysis, we extend the baseline model to incorporate endogenous labor and savings decisions, borrowing constraints, and a more comprehensive fiscal policy. This extension aims to create an empirically grounded economy that reflects existing inequality and policy instruments, enabling us to quantify both the effectiveness of carbon taxes in addressing climate externalities and their distributional impacts.

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[7] In the next section, household permanent labor endowments are replaced by sequences of uninsurable, idiosyncratic labor productivity shocks. Properly formulating the Negishi optimal tax in that environment would require the Negishi weights to be time-varying.

Before proceeding, it is important to note that these model features introduce *pecuniary* externalities wherein the decentralized equilibrium fails to deliver the optimal levels of savings and labor, as in Davila et al. (2012). In Appendix C, we fully characterize the constrained-efficient policies that correct these distortions alongside the climate externality. We demonstrate that the optimal carbon tax formulas derived in Section 4 continue to apply in this more general setting. Moreover, we solve for the constrained-efficient allocation and find that the resulting optimal carbon tax path is quantitatively similar regardless of whether these pecuniary externalities are addressed via optimal income taxes or left uncorrected. Consequently, the analysis in this section focuses on climate policies, taking the existing U.S. tax structure as given.

5.1 Quantitative Model

In this version of the economy, households face idiosyncratic labor productivity risk, which is uninsurable. We assume that labor productivity ε_t^i follows a Markov process with transition matrix $\pi\left(\varepsilon_t^i, \varepsilon_{t+1}^i\right)$. Households supply $n_t^i\varepsilon_t^i$ efficiency units of labor, where n_t^i denotes hours worked.

There is no aggregate uncertainty. Households can save in the form of real capital, k_{t+1}^i, which depreciates at a constant rate, δ_k. We assume households face the borrowing constraint

$$k_{t+1}^i \geq \underline{a}, \tag{32}$$

for every period t.

Households face capital income taxes, τ_{kt}, and a nonlinear labor income tax, $T_t^n(w_t\varepsilon_t^i n_t^i)$. In addition, they pay a carbon tax, τ_t^i, and receive lump-sum transfers, T_t^i. Thus, the household's budget constraint for every period t is

$$p_t(1+\tau_t^i)c_{dt}^i + c_{ct}^i + k_{t+1}^i \leq w_t\varepsilon_t^i n_t^i - T_t^n(w_t\varepsilon_t^i n_t^i) + (1-\tau_{kt})r_t k_t^i + (1-\delta_k)k_t^i + T_t^i \tag{33}$$

where w_t is the wage per efficiency unit of labor. The price of the clean good is normalized to one.

The household's problem is to choose consumption, labor, and savings to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t[u(c_{ct}^i, c_{dt}^i) - v(n_t^i) - x(S_{t+1})] \tag{34}$$

subject to (32) and (33).

The production of the clean and the dirty consumption good uses labor and capital as inputs according to a constant return to scale technology. Aggregate production of the clean and the dirty good is given by $Y_{ct} = F(K_{ct}, N_{ct})$ and $Y_{dt} = F(K_{dt}, N_{dt})$, respectively. The problem of the producer is to choose $\{\{N_{jt}, K_{jt}\}_{j=c,d}\}_{t=0}^{\infty}$ to maximize profits.

The government collects taxes and uses the proceeds to finance government spending and provide transfers, so that

$$\sum_i \mu_i \left(\tau_t^i p_t c_{dt}^i + T_t^n(w_t \varepsilon_t^i n_t^i) + \tau_{kt} r_t k_{t+1}^i\right) = G_t + \sum_i \mu_i T_t^i. \tag{35}$$

Finally, market clearing for each period t requires that

$$N_{ct} + N_{dt} = \sum_i \mu_i \varepsilon_t^i n_t^i \tag{36}$$

$$K_{ct} + K_{dt} = \sum_i \mu_i k_t^i \tag{37}$$

$$\sum_i \mu_i \left(c_{ct}^i + k_{t+1}^i - (1-\delta_k) k_t^i\right) + G_t = F(K_{ct}, N_{ct}) \tag{38}$$

$$\sum_i \mu_i c_{dt}^i = F(K_{dt}, N_{dt}) \tag{39}$$

Definition 3 *A competitive equilibrium with taxes* $\{\tau_t^i, \tau_{kt}, T_t^n, T_t^i\}_{t=0}^{\infty}$ *is a sequence of prices* $\{p_t, w_t, r_t\}_{t=0}^{\infty}$ *and allocations* $\{c_{ct}^i, c_{dt}^i, n_t^i, k_t^i, \{N_{jt}, K_{jt}\}_{j=c,d}\}_{t=0}^{\infty}$ *such that* (i) *given prices, households choose* $\{c_{ct}^i, c_{dt}^i, n_t^i, k_t^i\}_{t=0}^{\infty}$ *to maximize (34) subject to (33) and (32) for all i;* (ii) *profit maximizing prices are* $w_t = F_{Njt}$, $r_t = F_{Kjt}$ *for* $j = c, d$ *and* $p_t = 1$; (iii) *the stock of atmospheric carbon evolves according to* (1); (iv) *the government budget constraint* (35) *is satisfied; and* (v) *markets clear,* (36)–(39).

We use the model to study the implications of implementing a carbon tax in an empirically motivated economy designed to reflect existing inequality and taxes. The model economy targets moments of the US economy and its tax system and scales up emissions to account for the global carbon stock. We describe the calibration strategy below, summarized in Table 3.

5.2 Calibration

Economic Parameters. Because the distribution of consumption is a key determinant of the uniform constrained-efficient tax formula, the model should generate empirically relevant

Table 3: Calibration

Parameters	Values	Targets / Source
Preferences		
Discount factor, β	0.97	capital-to-output: 4.8
Risk aversion, κ	2	standard value
Disutility from labor, ϕ	20	average hours: 30 percent
Frisch elasticity, $1/\nu$	0.50	standard value
Climate		
Carbon absorption, δ	1/300	average life of carbon: 300 years
Carbon intensity, υ	326	1.4 degree increase by 2100 under BAU
Utility loss, ψ	0.03	welfare loss from 2.5 degree increase $\approx$ 1.7 percent output reduction
Clean share, γ	0.98	\$50/ton carbon tax leads to 0.8 degree reduction from BAU
Nonhomotheticity, $\bar{c}$	0.30	emissions intensity for low-income: 1.5 times higher than high-income households
Taxes		
Average, τ_n	0.23	average net tax rate: 13 percent
Progressivity, ν_y	0.19	37.9 percent marginal tax rate on top 1 percent earner
Capital, τ_k	0.27	Carey and Rabesona (2002)
Technology and shocks		
Capital weight, α	0.36	capital income share: 36 percent
Capital depreciation rate, δ_k	0.05	standard value
Persistence of wage process, ρ	0.94	author estimates
Standard deviation, σ_ε	0.21	Gini coefficient of earnings: 0.47
Superstar productivity, ε_{sup}	164	wealth share of top 1%: 34%
Persistence of superstar state, $\pi_{8,8}$	0.82	Gini coefficient of wealth: 0.83
Superstar probability, $\pi_{1:7,8}$	6e-5	fraction of superstars: 0.1%

levels of inequality. To achieve this, we employ a common strategy from the literature and include a superstar state in the Markov chain for the productivity process (Castaneda et al., 2003). To calibrate this Markov chain, we first approximate an AR(1) process (in logs) using the Rouwenhorst method (Kopecky and Suen, 2010) with nine normal (i.e., non-superstar) states. The persistence of the process for these states is set to 0.94 as measured in the PSID. Next, we jointly calibrate the standard deviation of the normal process, the value of superstar productivity, and the persistence of the superstar state to target three additional moments from the data: a Gini coefficient of earnings of 0.47, a top 1 percent wealth share of 0.34, and a Gini coefficient of wealth of 0.83. The probability of becoming a superstar from any normal state is set so that superstars account for 0.1 percent of the population. When a household exits the superstar state, its new productivity level is drawn from the ergodic distribution over the normal states as in Boar and Midrigan (2022). We set the borrowing limit to $\underline{a} = 0$ so that the model generates a share of households with non-positive wealth that is similar to that in the data.

The utility function is as specified in (26), and its preference parameters are calibrated to reflect the greater carbon intensity in the consumption baskets of low-income households as documented in Section 2. Accordingly, the nonhomotheticity parameter, $\bar{c}$, is set so that emissions intensity is 1.5 times higher for households in the bottom 10 percent of income compared to those in the top 10 percent. We assume the disutility of labor takes the form

$$v(n) = \phi \frac{n^{1+\nu}}{1+\nu}, \tag{40}$$

where ϕ and ν govern the disutility of labor and the Frisch labor elasticity, respectively.

Climate Parameters. We follow Golosov et al. (2014) in assuming that the stock of atmospheric carbon affects temperature changes according to:

$$T_t = \frac{\lambda}{\log(2)} \log\left(\frac{S_t}{\bar{S}}\right), \tag{41}$$

where $\bar{S} = 581$ represents the pre-industrialization carbon stock (in gigatons) and $\lambda = 3$, which implies that for each doubling of the carbon stock the temperature increases by 3 degrees (Celsius). We set $S_{2023} = 785$ to match the temperature rise of 1.3 degrees from the pre-industrial mean.

The carbon disutility cost takes the form

$$x(S) = \frac{\Psi}{2} S^2, \tag{42}$$

which is equivalent to the formulation in Barrage (2020) when risk aversion κ is equal to 2.[8] We calibrate Ψ so that the welfare loss associated with a 2.5-degree temperature increase is equivalent to that from a 1.74 percent decline in output, which combines the production and utility damages used in Barrage (2020).

We calibrate consumption carbon content, υ, so that under a business-as-usual scenario, there is an additional 1.4 degree increase in temperature from 2023 to 2100 (for a total of a 2.7 degree increase from pre-industrial levels).[9] We set the rate of natural reabsorption, δ, to 1/300 so that the average life cycle of carbon is 300 years (Archer 2005). Finally, we set the dirty consumption share in the utility function, $1-\gamma$, so that a \$50/ton carbon tax leads to a 0.8 degree reduction in global temperature from business-as-usual, consistent with Krusell and Smith Jr. (2022).

Taxes. We take the US tax structure as given, including its progressive earnings tax and transfer system and capital income tax. Thus, the BAU scenario corresponds to the US economy with its current tax regime and no carbon taxation. Following Heathcote et al. (2017) and Holter et al. (2019), the earnings tax bill for a household with pre-tax earnings $y = w_t n_t^i \varepsilon_t^i$ takes the form

$$T_t^n\left(y\right) = y - \tilde{y}^{\nu_y} \frac{1-\tau_n}{1-\nu_y} y^{1-\nu_y}, \tag{43}$$

where $\tilde{y}$ denotes average earnings in the economy. The parameter τ_n shifts the average tax rate, while ν_y controls the progressivity of the tax schedule. A flat labor income tax, τ_n, corresponds to $\nu_y = 0$. As ν_y increases, the tax function becomes more progressive. The capital income tax, τ_k, is set to the average rate for the US (Carey and Rabesona, 2002).

5.3 Model validity

In this section, we evaluate the fit of the model through both targeted and non-targeted moments. The goal is to assess whether we can plausibly use the model as a laboratory to run counterfactual policy experiments. Table 4 shows that in addition to matching all of the targeted moments, the calibration performs quite well for moments that we did not target.

The model reproduces the differential emission intensities of high-income and low-income households, and also of high-wealth and low-wealth households. The model also generates a Gini coefficient for consumption that is close to the data, although the emissions Gini

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[8] In Appendix E, we explore an extension of the model to heterogeneous climate damages and discipline the sensitivity with respect to income using survey data. We find that the main results are robust to this alternative specification.

[9] See `https://climateactiontracker.org/global/cat-thermometer`.

in the model is a little lower than it is in the data. Finally, the model features a wealth concentration that is close to the data, slightly overstating the wealth share of the top 5 percent while somewhat understating that of the top 0.1 percent.

Table 4: Model and data

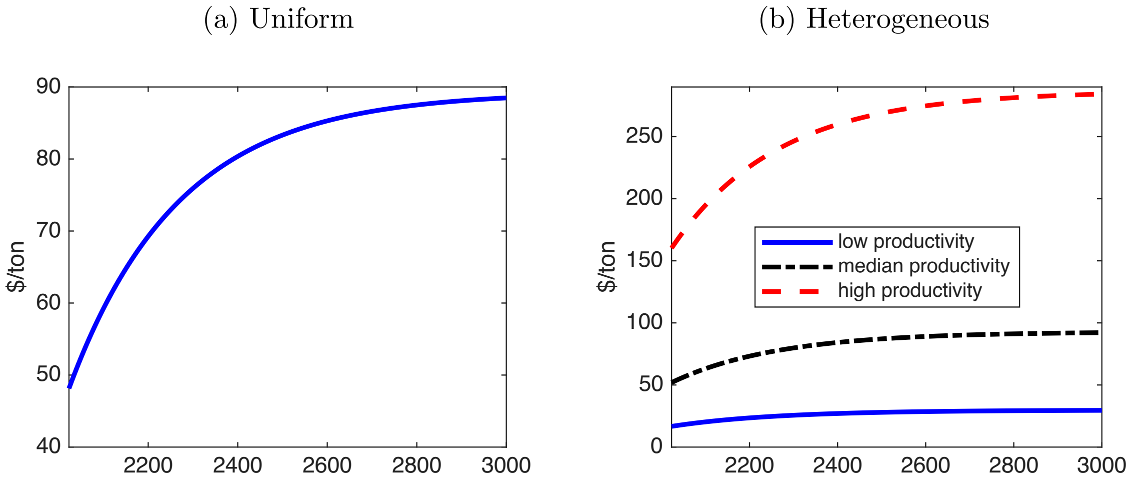
Targeted moments	Data	Model
Wealth-to-GDP	4.8	4.8
Temp. increase by 2100 under BAU	1.4	1.4
Temp. reduction from \$50 carbon tax	0.8	0.8
Welfare loss from 2.5° increase		
equivalent output reduction (percent)	1.7	1.7
Emissions intensity:		
bottom 10%/top 10% (income)	1.5	1.5
Gini coefficients:		
wealth	0.83	0.83
earnings	0.45	0.45
Wealth share of top 1%	0.35	0.35
Nontargeted moments		
Emissions intensity:		
bottom 10%/top 10% (wealth)	1.5	1.5
bottom 25%/top 25% (income)	1.3	1.3
Gini coefficients:		
consumption	0.31	0.29
emissions	0.30	0.20
Wealth share:		
top 5%	0.61	0.62
top 0.1%	0.15	0.07

5.4 Quantitative Results

We use the calibrated model to quantify the aggregate and distributional effects of different carbon tax policies on both the economy and the climate. The carbon tax formulas we derived in Section 4 are the theoretical foundation for our quantitative analysis. Specifically, we evaluate the formulas for the heterogeneous tax rates (24) and the uniform tax rate (28) at the competitive equilibrium allocation to determine the path of the carbon taxes. This requires searching for a fixed point in the space of carbon tax sequences through a simple iterative strategy. Starting from a sequence with zero carbon taxation (i.e., business-as-usual), we compute the equilibrium paths for the carbon stock, the wealth distribution, and household consumption decisions. Next, we plug this allocation into the carbon tax formula to generate a new sequence of carbon taxes. Then we solve for the transition corresponding to this updated sequence and evaluate the tax formula again. We continue to iterate on this procedure, repeatedly updating the path of carbon taxes until it converges. Unless otherwise noted, the carbon tax policy rebates each household's carbon tax bill back to it.

We begin by contrasting the outcomes of two carbon tax policies: one in which the government levies the same flat rate, τ_t, on all households according to the uniform constrained-efficient carbon tax formula in (28), and a second one, in which the carbon tax schedule places higher tax rates on more productive households as in (24).[10] Figure 2 plots the time path of the uniform and heterogeneous optimal carbon tax schedules. In both cases, the tax rates rise over time, reflecting that the greatest climate damages appear in the future and are thus heavily discounted in the initial periods. As time passes, however, and atmospheric carbon levels rise, the benefits of discouraging additional carbon emissions become more pressing.

Figure 2: Constrained-efficient carbon tax


(a) Uniform

(b) Heterogeneous

Along the uniform tax path (shown in panel a), the carbon tax rate starts at \$48/ton and gradually increases to a long-run value of \$89/ton. When the carbon tax schedule differentiates households by labor productivity (effectively a household's hourly wage), the tax rates display a significant degree of variation. The tax rate on the lowest productivity households rises from \$17/ton in the initial period to \$30/ton in the long run. In contrast, the analogous rates for households with the highest non-superstar productivity are \$160/ton

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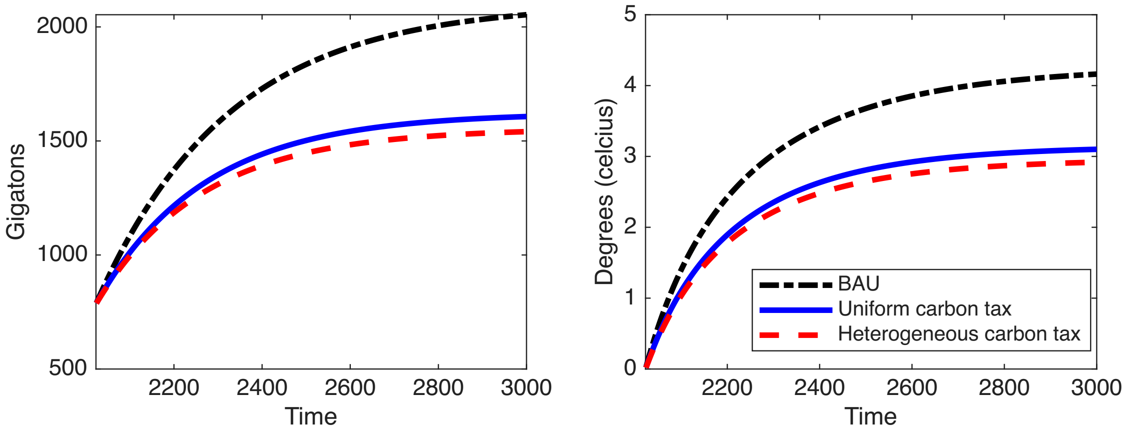
[10] We index the heterogeneous tax rates by productivity. While taxes could in principle depend on the entire history of a household's productivity shocks, this is computationally infeasible. Alternatively, indexing by wealth would introduce a distortion to the savings decision.

and \$286/ton. The wide difference in tax rates reflects the sizeable variation in the marginal utility of consumption across households with different labor productivity. It is not surprising then that the carbon tax rate for superstar households is extraordinarily high, starting at \$2,490/ton and rising to nearly \$4,430/ton.

Because we have shut off the wealth effect by rebating back each household's carbon tax payment, the aggregate levels of labor, capital, consumption, and output remain very close to their initial values under either policy. The composition of these aggregates between dirty and clean goods, however, does change since the tax distorts each household's optimal consumption bundle toward a higher share of clean consumption.

Turning to the climate, global temperature still rises under either carbon tax policy. Nevertheless, relative to the BAU scenario, both policy interventions have a substantial effect on the evolution of the carbon stock and global temperature over time (Figure 3). The greatest temperature moderation is produced by the productivity-indexed carbon tax. It reduces global temperatures by 0.5°C relative to the BAU path over 100 years and by 1.1°C over 500 years.

Figure 3: Carbon and temperature


(a) Carbon stock

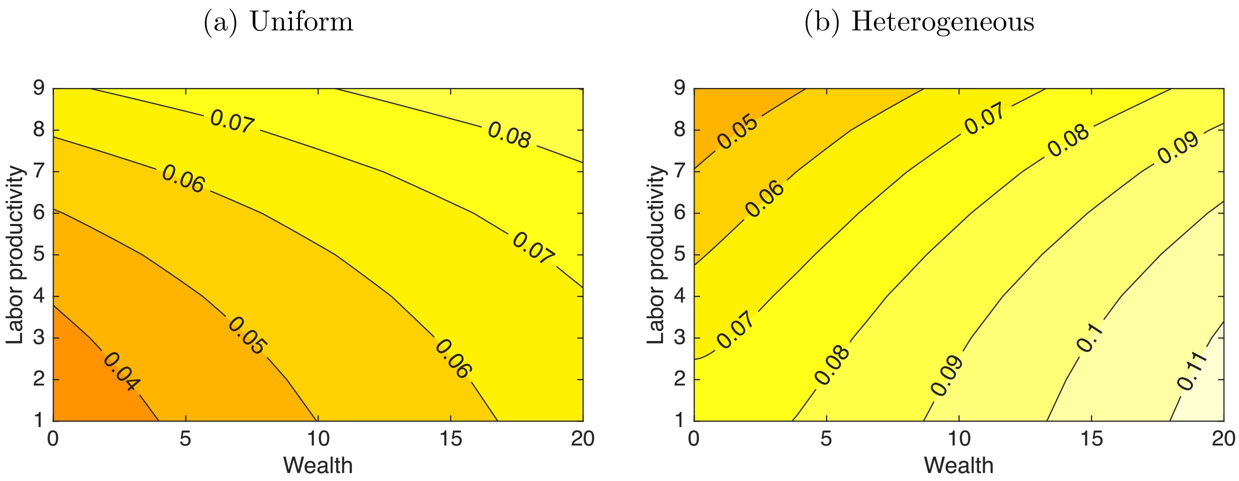
(b) Temperature increase

Next, we compute the change in welfare from undergoing the policy-induced transitions relative to the BAU baseline and examine the differential effects of the carbon tax across the wealth and income distribution. The welfare change is measured from the perspective of agents at $t = 1$ and includes the entire discounted sequence of utility differences.

Figure 4 displays the change in welfare for households across the wealth and productivity distribution under carbon taxation. The wealth levels shown cover 98 percent of households.

In panel (a), where carbon taxes are uniform, all households experience welfare gains, with the largest gains accruing to the most productive households with high wealth. While all households benefit from reduced emissions, the costs, particularly from distortions in consumption patterns, disproportionately affect poorer households. This is further illustrated in the heterogeneous tax case (panel b), where some of these distortions are shifted from low- to high-productivity households. In this case, welfare gains are decreasing in productivity, for a fixed level of wealth. In fact, a small fraction of the highest-productivity households initially suffer a small welfare loss due to the steep taxes imposed on their consumption of carbon-intensive goods.

Figure 4: Welfare

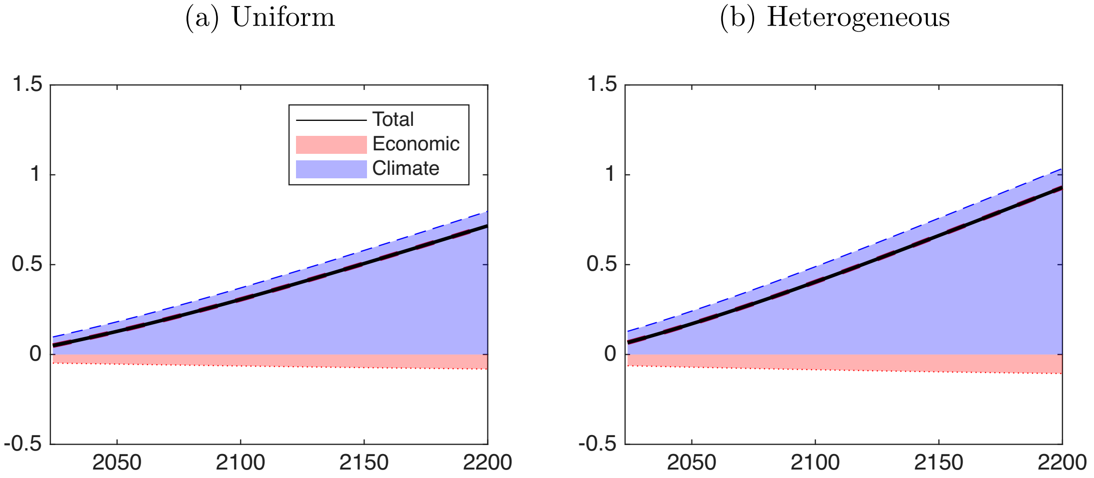

(a) Uniform

(b) Heterogeneous

Notes: Wealth is in units of per capita GDP. Labor productivity corresponds to the nine discrete states of the persistent idiosyncratic productivity process. Welfare gains are expressed in consumption equivalents (percent) relative to the Business-as-Usual (BAU) scenario.

There is a timing mismatch between the costs and benefits of taxing carbon emissions. The consequences of unmitigated carbon emissions build up over time, with the most severe effects from the business-as-usual scenario materializing in the distant future. Meanwhile, any carbon policy stringent enough to have a meaningful impact on the path of the carbon stock must impose immediate costs on households. The balance between these costs and benefits shifts over time. To illustrate, we compute average welfare over time by integrating welfare changes from time t forward against the income and wealth distribution. For each period t_n, $n = 1, ..., T$, this measures the permanent increase in consumption that households require to be indifferent between being dropped into period t_n of the BAU transition path or being dropped into t_n on the climate policy transition path (i.e., where a carbon tax was enacted starting in $t = 1$). Figure 5 plots the evolution of average welfare, measured as consumption equivalents under the veil of ignorance, and decomposes welfare changes into economic and climate factors. Although we do not explicitly model overlapping generations, one could interpret this as a measure of the average welfare of future generations. Over time, the benefits from a better climate grow, while the costs associated with consumption distortions remain relatively stable.

Figure 5: Average welfare over time



(a) Uniform

(b) Heterogeneous

Units: percent.

Table 5 is the quantitative model analogue of Table 2 from the simple model, and it reports the initial carbon tax, climate consequences summarized as the long-run reduction in temperature, average initial welfare gain, and support for the policy. As in the simple model, the uniform carbon tax (lump-sum rebated back to households) delivers slightly lower average welfare gains than the heterogeneous carbon tax. Both policies have a very high amount of support. The uniform carbon tax is Pareto improving relative to BAU, while in the heterogeneous case all households benefit except some superstars.

We also report in Table 5 the outcome of three additional policies. The first imposes a uniform carbon tax and then divides the revenue between financing a subsidy on clean goods and a uniform lump-sum transfer as described in Corollary 1 (shown in row 3). The welfare and climate consequences under this policy are virtually identical to those of the uniform carbon tax with individual rebates (row 2). While Corollary 1 establishes an equivalence in the simple model, this result is not guaranteed to hold in the more general quantitative model studied in this section. This is an important result, since the all-uniform tax/subsidy/transfer policy does not require the policymaker to have knowledge of each household's income or wealth, making it easier to implement.

Table 5: Welfare and Climate

Policy	Initial carbon tax ($/ton)	Long-run temp. reduction (degrees)	Average welfare gain (percent)	Support (percent)
Heterogeneous tax with rebate	59.4*	1.3	0.07	99.98
Uniform tax with rebate	48.1	1.1	0.05	100.00
Uniform tax & subsidy/transfer	48.1**	1.1	0.05	100.00
Uniform tax and subsidy	48.1**	1.1	0.01	53.03
Uniform tax and transfer	48.1	1.1	0.12	93.97

Note: * denotes the average carbon tax rate, weighted by dirty consumption expenditures. ** denotes the effective carbon tax rate, defined as $(1 + \tau_{dt})/(1 - \tau_{ct}) - 1$.

The last two rows in Table 5 report the case where the carbon tax revenue is used to entirely finance either a clean subsidy or a uniform lumpsum transfer. Note that in contrast to the constrained-efficient taxes studied earlier, these climate policies imply some resource transfers across households. When the carbon tax revenues finance a clean subsidy only (row 4), the average welfare gain and support are substantially lower than when they finance both a clean subsidy and a uniform lumpsum transfer (row 3). Because the spending of poorer households is more carbon intensive, a carbon tax/clean subsidy shifts resources from high marginal-utility households to low marginal-utility households, which reduces the average welfare gain. Among the policies considered, the highest average welfare gain is achieved by returning all carbon tax revenues as a uniform lumpsum transfer (row 5). Support for this policy is not universal, however, as the highest income and wealth households oppose it. The exercises highlight the importance of our theory to informing carbon tax design. Deviations prove costly, either by drastically diminishing aggregate welfare (in the case of clean subsidies) or by eroding universal support (in the case of pure lump-sum transfers).

5.5 Robustness and Heterogeneous Damages

Evidence suggests that low-income households may be disproportionately vulnerable to climate change, implying that the damage function itself potentially varies across the income distribution. To address this, we extend the model to allow for heterogeneous climate damages, where the utility cost of climate change is higher for the poor. Specifically, we introduce an income elasticity of climate damages that we discipline using survey data on willingness-to-pay for climate mitigation. We find that our core result is robust: even when the poor bear a disproportionate burden of climate damages, the uniform carbon tax with rebates remains

Pareto improving. In fact, because the poor value the climate benefits of the tax more highly in this scenario, the welfare gains at the bottom of the distribution are amplified.

Finally, we perform sensitivity analysis regarding our preference parameters. We find that the Pareto improvement holds across alternative values for risk aversion and the discount factor. We detail these exercises in Appendix E.

6 Concluding Remarks

In this paper, we study the design of optimal climate policy in the presence of economic inequality. Our analysis yields several key theoretical insights into the implications of incorporating inequality into climate policy design: (*i*) a heterogeneous carbon tax emerges as a progressive natural solution, where higher-income households face higher tax rates; (*ii*) if uniformity of the carbon tax is desired, the carbon tax is lower than that prescribed under a representative agent framework; and (*iii*) uniform taxation can be achieved using a combination of three instruments: a uniform carbon tax, a uniform clean energy subsidy, and a uniform lump-sum transfer. Our quantitative analysis finds that the uniform climate policy is welfare-improving for every household.

Our work suggests several promising directions for future research. First, while this paper focuses on consumption inequality, climate inequality may be linked to employment in sectors that are more exposed to climate change. Endogenizing the unequal effects of climate change by enriching the modeling of the economy's productive sectors presents a significant avenue for further research. Second, households' climate inequality stems also from heterogeneous climate impacts, with the concern that low-income households might be more vulnerable to climate shocks. While we show that our results remain robust when utility costs are heterogeneous by income, further research quantifying this aspect of climate vulnerability would be beneficial. Finally, while this paper examines within-country income and wealth inequality, multiple dimensions of inequality are crucial to the climate problem, including cross-country inequality and intergenerational inequality. Exploring the policy implications of these alternative dimensions would provide valuable insights for climate policy design.

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A Data Appendix

A.1 Details on dataset construction

We combine data on household expenditures, income, and liquid wealth from the Consumer Expenditure Survey (CEX) with data on embodied emissions from the Environmental Protection Agency (EPA). The emissions data include carbon dioxide (CO_2) emissions and other greenhouse gases such as methane and nitrous oxide, converted to CO_2 equivalents using the Intergovernmental Panel on Climate Change (IPCC) Assessment Report's 100-year global warming potential. It covers supply chain emissions (from cradle to factory gate) and margins (from factory gate to shelf, including transportation, wholesale, and retail).

To combine the expenditure data with the emissions data, we first construct a concordance to map 671 Universal Classification codes (UCC) of CEX expenditures to 394 North American Industry Classification System (NAICS) codes used in the emissions dataset (EPA). For example, the UCC code 100210 (cheese) is linked to the NAICS-6 code 311513 (cheese manufacturing), which is associated with 1.6 kilograms of CO_2-equivalent embodied emissions per 2018 dollar spent. As another example, the UCC code 560110 (physician services) corresponds to the NAICS-4 code 6211 (offices of physicians), which is associated with 0.1 kilograms of CO_2-equivalent embodied emissions per dollar.[11]

The CEX microdata consist of two surveys: The diary survey collects detailed information on a subset of household expenditures (especially for groceries, such as flour, rice, and white bread) for two consecutive weeks and the interview survey collects more spending categories that cover most household expenditures (e.g., food at home, college tuition, camping equipment, and airline fares) for one year. Though the two surveys are not linked, we use the detailed food and beverage expenditures from the diary survey to estimate an embodied emission function and apply it to the interview data on food and beverages at home. For all other interview expenditure categories, we use the constructed UCC-NAICS concordance to directly calculate embodied emissions. Finally, we include direct tailpipe emissions, which are associated with about 9 kilograms of CO_2 per gallon driven (EPA).[12]

In addition to showing how emission intensities relate to income and wealth in Section 2, here we regress the intensities on the natural logs of income and wealth, shown in Table A.1. Columns (1)–(2) demonstrate that wealth and income are negatively associated with embodied emission intensities, statistically significant at the 1 percent level. Column (3) shows that this result is robust to controlling for education, age, and family size fixed effects. These effects are also economically significant: Using the coefficients in column (3), increases of one standard deviation in log income and wealth are associated with increases of 2.1 and 6.2 percentage points in the embodied emission intensities.

Table A.1: Embodied emission intensity

	(1)	(2)	(3)
Wealth	-2.48^{***}		-1.87^{***}
	(0.131)		(0.175)
Income		-4.70^{***}	-2.14^{***}
		(0.243)	(0.612)
Observations	1488	5102	1488
Adjusted R^2	0.195	0.068	0.241

Standard errors in parentheses. (3) additionally includes college, age, and family size fixed effects. *** represents statistical significance at the 1 percent level.

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[11] The full concordance is provided online.

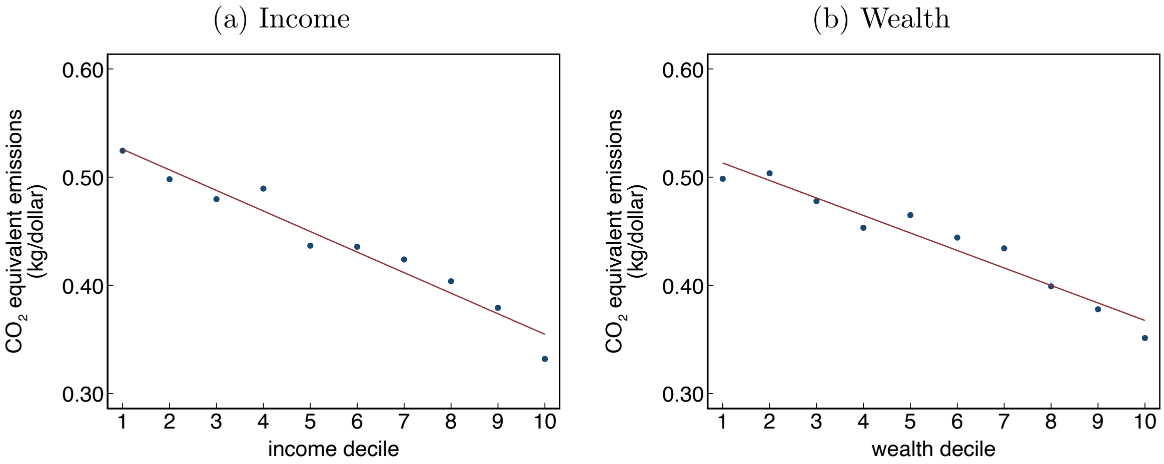
[12] See `https://www.fueleconomy.gov/feg/label/calculations-information.shtml`.

A.2 Panel Study of Income Dynamics

We redo the empirical analysis using household data from the Panel Study of Income Dynamics (PSID). Compared to the CEX, the PSID has the advantage that it contains a more complete representation of household wealth, including financial and nonfinancial assets and debt. On the other hand, the PSID expenditure data are more aggregated, compared with the CEX. Thus, the PSID analysis presented here complements our CEX analysis and provides a useful robustness exercise.

We combine the PSID expenditure data with the EPA emissions data, similar to the way described above. As shown in Figure A.1, the embodied emissions intensity is decreasing in both income and in wealth. Thus, we confirm that the patterns documented using the CEX are similar to those using the PSID.

Figure A.1: Embodied emissions


(a) Income

(b) Wealth

B Competitive Equilibrium in the Simple Model

In a decentralized environment, households are endowed with ε_i units of labor (inelastically supplied) and choose consumption to maximize utility (2) subject to the following set of budget constraints,

$$p_t(1+\tau_t^i)c_{dt}^i + c_{ct}^i \leq w_t\varepsilon^i + T_t^i, \tag{44}$$

for every period t, where p_t is the relative price of dirty to clean consumption, w_t is the wage, and (τ_t^i, T_t^i) are a carbon tax and a lump-sum transfer.

The government collects carbon taxes and uses the proceeds to finance lump-sum transfers to households. The budget constraint of the government for every period t is

$$\sum_i p_t\tau_t^i\mu_i c_{dt}^i = \sum_i \mu_i T_t^i. \tag{45}$$

There are two production units, the clean and the dirty good producers indexed by j. A representative firm uses labor as the only input in each sector according to a linear technology. Thus, the aggregate production of the clean and the dirty good is given by $Y_{ct} = N_{ct}$ and $Y_{dt} = N_{dt}$, respectively.

Finally, market clearing for each period t requires that

$$N_{ct} + N_{dt} = \sum_i \mu_i\varepsilon^i \tag{46}$$

$$N_{ct} = \sum_i \mu_i c_{ct}^i \tag{47}$$

$$N_{dt} = \sum_i \mu_i c_{dt}^i \tag{48}$$

Definition 4 (Competitive Equilibrium with Carbon Taxes) *A competitive equilibrium with taxes* $\{\tau_t^i, T_t^i\}_{t=0}^{\infty}$ *is a sequence of prices* $\{p_t, w_t\}_{t=0}^{\infty}$ *and allocations* $\left\{\{c_{jt}^i, N_{jt}\}_{j=c,d}\right\}_{t=0}^{\infty}$ *such that* (*i*) *given prices and taxes, households choose* $\{c_{ct}^i, c_{dt}^i\}_{t=0}^{\infty}$ *to maximize* (2) *subject to* (44) *for all* i*;* (*ii*) *given prices, firms of sector* $j = \{c, d\}$ *choose* $\{N_{jt}\}_{t=0}^{\infty}$ *to maximize profits;* (*iii*) *the government budget constraint* (45) *is satisfied;* (*iv*) *the stock of atmospheric carbon evolves according to* (1)*, and* (*v*) *prices clear the markets.*

At an interior solution, household and firm optimality conditions imply:

$$\frac{u_{dt}^i}{u_{ct}^i} = 1 + \tau_t^i \tag{49}$$

which states that the marginal rate of substitution between clean and dirty consumption equals the relative price for every period t. In this economy, profit maximization on the firm's side implies that $p_t = w_t = 1$ in every period t.

C Constrained Efficiency in the Quantitative Model

In this section, we show that the carbon tax formulas derived in the simple economy in Section 4 extend to the richer quantitative economy in Section 5. It is not obvious that this should be the case since incomplete market economies are typically not constrained-efficient. This is because the presence of uninsurable idiosyncratic risk distorts households' labor and savings decisions relative to the complete markets environment. In general equilibrium, this leads to inefficient pricing as households do not internalize their effect on aggregates. As shown in Davila et al. (2012), these pecuniary externalities can be corrected by a utilitarian planner without transferring resources across households, and this constrained-efficient allocation can be decentralized through a system of household-specific taxes and rebates.

In our quantitative economy, these pecuniary externalities coexist with the climate externality. A constrained utilitarian planner would correct all externalities jointly, but the formulas in Section 4 address the climate externality in isolation. Does the climate policy under the planner's joint solution coincide with our optimal formulas?

Consider the model environment laid out in Section 5. The *constrained-efficient allocation* is the sequence $\{\{c^i_{ct}, c^i_{dt}, n^i_t, k^i_{t+1}\}_i, \{K_{jt}, N_{jt}\}_{j=c,d}, S_t\}_{t=0}^{\infty}$ that solves the social planner's problem, which is to maximize

$$\mathbf{E}_0 \sum_i \mu_i \left[\sum_{t=0}^{\infty} \beta^t \left(u(c^i_{ct}, c^i_{dt}) - v(n^i_t) - x(S_{t+1}) \right) \right], \tag{50}$$

subject to the carbon cycle (1), household borrowing constraints (32), and implementability conditions

$$c^i_{dt} + c^i_{ct} + k^i_{t+1} \leq F_{Nt}\varepsilon^i_t n^i_t + F_{Kt}k^i_t + (1-\delta_k)k^i_t. \tag{51}$$

The implementability conditions can be derived by substituting equilibrium prices $w_t = F_{Nt}$, $r_t = F_{Kt}$, and $p_t = 1$, and the constraint that individual taxes must be rebated lump-sum into each household's budget constraint (33), with $G_t = 0$. The constrained-efficient allocation takes the market structure and individual constraints as given, and excludes net transfers across households.

The first-order conditions for the constrained-efficient social planner's problem are:

$$(c_{dt}^i) : \mu_i u_{dt}^i - \upsilon \mu_i \sigma_t - \mu_i \lambda_t^i = 0 \tag{52}$$

$$(c_{ct}^i) : \mu_i u_{ct}^i - \mu_i \lambda_t^i = 0 \tag{53}$$

$$(n_t^i) : -\mu_i v_{n_t}^i + \mu_i \lambda_t^i F_{Nt} \varepsilon_t^i + \sum_j \mu_j \lambda_t^j (F_{NNt} \mu_i \varepsilon_t^i \varepsilon_t^j n_t^j + F_{KNt} \mu_i \varepsilon_t^i k_t^j) = 0 \tag{54}$$

$$(k_{t+1}^i) : -\mu_i \lambda_t^i + \mu_i \phi_t^i + \beta \mu_i \mathbf{E}_t \left[\lambda_{t+1}^i (1 - \delta_k) + \lambda_{t+1}^i F_{K,t+1} \right] \tag{55}$$

$$+ \beta \mathbf{E}_t \left[\sum_j \mu_j \lambda_{t+1}^j (F_{NK,t+1} \mu_i \varepsilon_{t+1}^j n_{t+1}^j + F_{KK,t+1} \mu_i k_{t+1}^j) \right] = 0$$

$$(S_{t+1}) : \sigma_t - x'(S_{t+1}) + \beta(1 - \delta)\sigma_{t+1} = 0 \tag{56}$$

where $\beta^t \sigma_t$, $\beta^t \mu_i \lambda_t^i$, $\beta^t \mu_i \phi_t^i$ are the Lagrange multipliers on the carbon cycle, implementability conditions, and household borrowing constraints, respectively.

For ease of exposition, we define the following objects:

$$\Lambda_{nt}^i \equiv -\frac{\sum_j \mu_j u_{ct}^j (F_{NNt} \varepsilon_t^j n_t^j + F_{KNt} k_t^j)}{u_{ct}^i F_{Nt}} \tag{57}$$

$$\Lambda_{kt}^i \equiv -\frac{\sum_j \mu_j u_{ct}^j (F_{NKt} \varepsilon_t^j n_t^j + F_{KKt} k_t^j)}{u_{ct}^i (F_{Kt} + 1 - \delta_k)} \tag{58}$$

Using these definitions, we can combine the first-order conditions to get the following constrained-efficient intratemporal wedge between consumption and leisure:

$$\frac{u_{nt}^i}{u_{ct}^i} = F_{Nt} \varepsilon_t^i (1 - \Lambda_{nt}^i) \tag{59}$$

where the last term on the right-hand side captures the pecuniary externality as the social planner internalizes that household's working and savings decisions affect equilibrium factor prices.

Similarly, the intertemporal wedge incorporates the effect of the individual's saving decision over the equilibrium interest rate, and the Euler equation is given by:

$$u_{ct}^i \geq \beta \mathbf{E}_t \left[u_{c,t+1}^i \left(F_{Kt+1} + 1 - \delta_k \right) \left(1 - \Lambda_{k,t+1}^i \right) \right], \tag{60}$$

which holds with equality if household i's borrowing constraint is not binding.

Finally, the intratemporal wedge for clean and dirty consumption is

$$\frac{u_{dt}^i}{u_{ct}^i} = 1 + \frac{\upsilon \sigma_t}{u_{c_t}^i} \tag{61}$$

with

$$\sigma_t = \sum_{j=1}^{\infty} [\beta(1-\delta)]^{j-1} x'(S_{t+j}). \tag{62}$$

Thus, the constrained-efficient allocation equalizes the marginal rate of substitution between clean and dirty goods with the relative social price that includes the climate externality, in addition to satisfying the intratemporal and intertermporal Euler equations.

Proposition 3 characterizes the constrained-efficient policy that implements the optimal allocation as a market equilibrium with taxes. To derive analytical expressions for the tax rates that implement the constrained-efficient allocation, we assume that $\underline{a}$ is sufficiently low such that the borrowing constraint is not binding. Furthermore, to implement the constrained-efficient allocation, we show that the tax rates for carbon, labor income, and capital income must be individual-specific.

Proposition 3 *Let* $x \equiv \{c^i_{ct}, c^i_{dt}, n^i_t, k^i_{t+1}, \{K_{jt}, N_{jt}\}_{j=c,d}, S_t\}^\infty_{t=0}$ *be the constrained-efficient allocation. Then,* x *is also a competitive market equilibrium with taxes given by*

$$\tau^i_t = \frac{\upsilon\sigma_t}{u^i_{c_t}} \tag{63}$$

$$\tau^i_{nt} = \Lambda^i_{nt} \tag{64}$$

$$\tau^i_{kt} = \Lambda^i_{kt} \tag{65}$$

The tax revenue is rebated back lump-sum.

The proof is in Appendix D. Proposition 3 shows that the carbon tax rule in the quantitative model adheres to the same formula as in Proposition 1. The actual optimal carbon tax rate, however, may differ from the rate computed in the quantitative analysis, as the calibrated economy is not constrained-optimal. Moreover, capital and labor income taxes are also part of the constrained-efficient policy. These taxes are necessary to internalize the pecuniary externalities stemming from idiosyncratic risk and market incompleteness. Notice that the constrained-efficient policy in Proposition 3 is history-dependent, just as in Davila et al. (2012).

Uniform Constrained-Efficient Taxes. As in the simple model, uniformity of the carbon tax—if desirable—must be added as an exogenous constraint. Restricting attention to preferences of the form (26), *the uniform constrained-efficient allocation* comes from maximizing (50) subject to the carbon cycle (1), the implementability constraint (51), borrowing constraint (32), and the additional constraint (27). The first-order necessary conditions for this social planner's problem are:

$$(c^i_{dt}) : \mu_i u^i_{dt} - \upsilon \mu_i \sigma_t - \mu_i \lambda^i_t + \sum_{j\neq i} \eta^{ij}_t \left(c^j_{ct} + \bar{c}\right) - \sum_{j\neq i} \eta^{ji}_t \left(c^j_{ct} + \bar{c}\right) = 0 \tag{66}$$

$$(c^i_{ct}) : \mu_i u^i_{ct} - \mu_i \lambda^i_t - \sum_{j\neq i} \eta^{ij}_t c^j_{dt} + \sum_{j\neq i} \eta^{ji}_t c^j_{dt} = 0 \tag{67}$$

$$(n^i_t) : -\mu_i v^i_{n_t} + \mu_i \lambda^i_t F_{Nt} \varepsilon^i_t + \sum_j \mu_j \lambda^j_t (F_{NNt} \mu_i \varepsilon^i_t \varepsilon^j_t n^j_t + F_{KNt} \mu_i \varepsilon^i_t k^j_t) = 0 \tag{68}$$

$$\begin{aligned} (k^i_{t+1}) : &-\mu_i \lambda^i_t + \mu_i \phi^i_t + \beta \mu_i \mathbf{E}_t \left[\lambda^i_{t+1}(1-\delta_k) + \lambda^i_{t+1} F_{Kt+1}\right] \\ &+ \beta \mathbf{E}_t \left[\sum_j \mu_j \lambda^j_{t+1} (F_{NKt+1} \mu_i \varepsilon^j_{t+1} n^j_{t+1} + F_{KKt+1} \mu_i k^j_{t+1})\right] = 0 \end{aligned} \tag{69}$$

$$(S_{t+1}) : \sigma_t - x'(S_{t+1}) + \beta(1-\delta)\sigma_{t+1} = 0 \tag{70}$$

where $\beta^t \eta^{ij}_t$ is the Lagrange multiplier on the constraint (27) and $\beta^t \sigma_t$, $\beta^t \mu_i \lambda^i_t$, $\beta^t \phi^i_t$ are the Lagrange multipliers on the carbon cycle, household implementability constraints, and borrowing constraints, respectively.

Let $x \equiv \{c^i_{ct}, c^i_{dt}, n^i_t, k^i_{t+1}, \{K_{jt}, N_{jt}\}_{j=c,d}, S_t\}^{\infty}_{t=0}$ be the solution to the uniform-constrained social planner's problem. Corollary 2 states that the constrained-efficient uniform carbon tax follows the rule in Proposition 2.

Corollary 2 *The carbon tax that implements x follows* (28)*.*

The proof is in Appendix D. As is the case in Proposition 3, the uniform carbon tax must be coupled with individual labor and capital income taxes/subsidies that follow (64) and (65) to fully implement the optimal allocation.

Corollary 3 *The uniform constrained efficient allocation can also be implemented with a set of uniform climate policies as specified in equation* (31)*.*

This provides the analogue of Corollary 1 for the uniform constrained efficient allocation in the quantitative model, providing a useful implementation with a set of uniform carbon taxes, uniform clean subsidies, and a uniform lumpsum transfer, in addition to using individual labor and capital income taxes. The proof is provided in Appendix D.

C.1 Quantitative Analysis

Making a direct comparison between our baseline quantitative analysis from Section 4 and the constrained efficient allocation is complicated by the presence of an exogenous level of government spending that is disproportionately financed by rich households. In the constrained efficient problem, it is not clear how to distribute this burden in an equivalent manner. To sidestep this issue, we follow Davila et al. (2012) by re-calibrating our economy to a steady state without labor or capital income taxes, government spending, or lump-sum transfers; effectively, eliminating the government in the initial steady state. Additionally, we remove the superstar earnings state which reduces the wealth tail relative to the empirically motivated model from Section 5.2. We refer to this economy as the *laissez-faire* economy. With the exception of moments related to US fiscal policy or wealth inequality, we keep the same targets as those from the model in Section 5.2. The parameters are summarized in Table C.2.

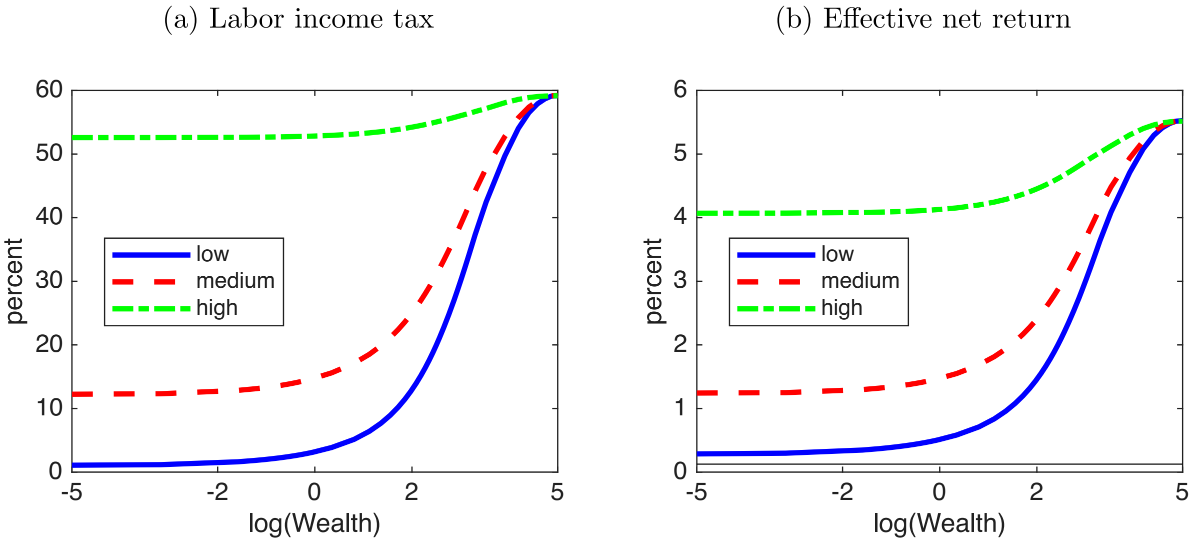
Table C.2: Calibration of the Laissez-Faire Economy

Parameters	Values	Targets / Source
Preferences		
Discount factor, β	0.95	capital-to-output: 4.8
Risk aversion, κ	2	standard value
Disutility from labor, ϕ	31	average hours: 30 percent
Frisch elasticity, $1/\nu$	0.50	standard value
Climate		
Carbon absorption, δ	1/300	average life of carbon: 300 years
Carbon intensity, υ	326	1.4 degree increase by 2100 under BAU
Utility loss, ψ	0.04	welfare loss from 2.5 degree increase $\approx$ 1.74 percent output reduction
Clean share, γ	0.98	\$50/ton carbon tax leads to 0.8 degree reduction from BAU
Nonhomotheticity, $\bar{c}$	0.15	emissions intensity for low-income: 1.5 times higher than high-income households
Technology and shocks		
Capital weight, α	0.36	capital income share: 36 percent
Capital depreciation rate, δ_k	0.05	standard value
Persistence of wage process, ρ	0.94	author estimates
Standard deviation, σ_ε	0.29	Gini coefficient of earnings: 0.47

We will compare the outcomes from two exercises. In the *carbon-tax-only* (CTO) exercise, we introduce a carbon tax into the laissez-faire economy. In the second one, both income taxes and a carbon tax jointly correct the climate and pecuniary externalities so as to implement the constrained-efficient allocation (CEA). By contrasting the resulting climate policies and environmental outcomes across these two cases, we can assess the extent to which pecuniary externalities affect the carbon tax formula.

Before turning to climate outcomes, we characterize the fiscal instruments required to implement the constrained-efficient allocation. As established in Proposition 3, the presence of uninsurable idiosyncratic risk creates a pecuniary externality that the planner corrects through labor and capital income taxes/subsidies. Figure C.2 plots these constrained-efficient schedules at the initial period. We find that the optimal labor income tax is progressive (panel a) and rises steeply in both productivity and wealth. At the same time, the planner aligns private intertemporal decisions with the social optimum by subsidizing savings, particularly for high-wealth and high-productivity households. This aligns with the "High Wealth Dispersion" economy analyzed in Davila et al. (2012), where the market equilibrium features an underaccumulation of capital relative to the constrained optimum. To correct this, the planner provides incentives for capital accumulation that increase with wealth, as these agents are the primary savers in the economy.

Figure C.2: Effective tax rates (CEA)


(a) Labor income tax

(b) Effective net return

Notes: Wealth is in units of per capita GDP.

Despite the massive difference in the treatment of labor and capital across the two economies, the equilibrium carbon tax schedules under the CEA and the CTO are quite similar. The two polices are compared in Figure C.3. Initially, the optimal carbon tax is lower in the CEA case ($26.4/ton initial) than in the CTO case ($32.7/ton initial). The CEA allocation requires higher levels of capital, leading to lower consumption initially. As a consequence, the social value of consumption is higher, which results in a more moderate carbon price. Over time, as the capital levels become larger in the constrained efficient allocation, the optimal carbon tax eventually exceeds that in the carbon-tax-only economy.

While the desire to accumulate capital shifts consumption in the CEA further out into the transition, both the level and shape of the carbon tax schedules are little changed, and the path of climate and of temperature are virtually identical in the two cases.

Figure C.3: Carbon tax comparison

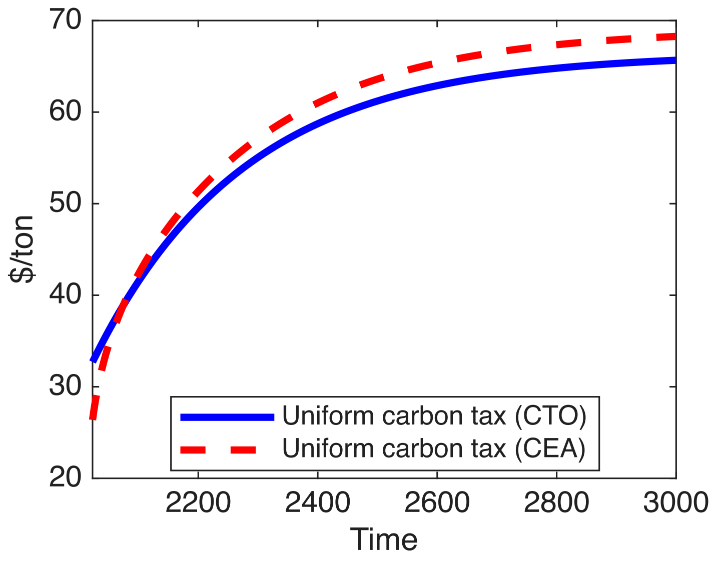


Figure C.4: Constrained-efficient carbon tax

(a) Uniform

(b) Heterogeneous

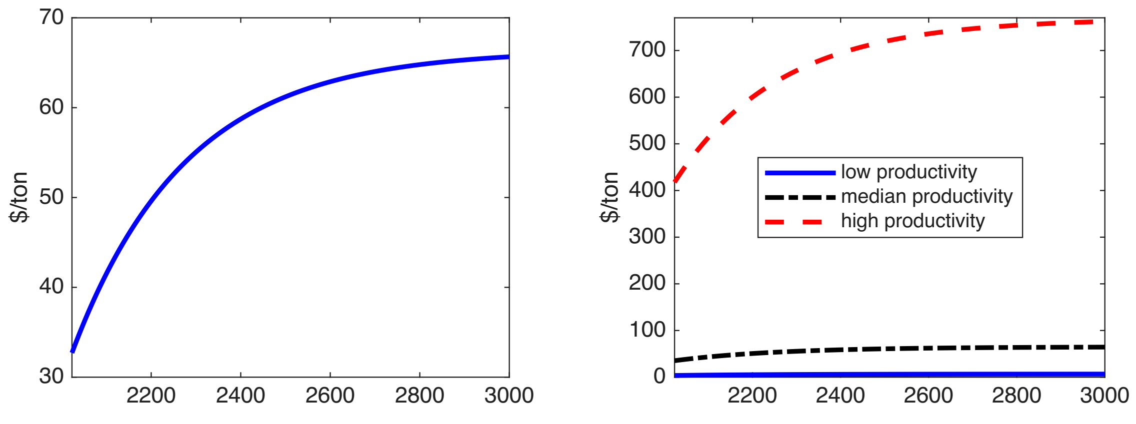
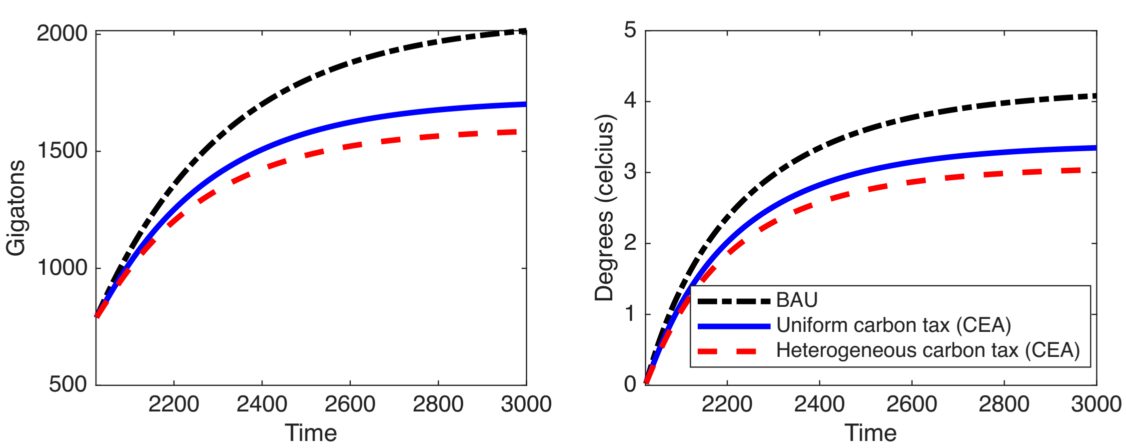


Figure C.4 displays the time path of the uniform and heterogeneous carbon taxes in the constrained-efficient economy. As in our baseline case, tax rates rise over time as the social cost of carbon increases, and the heterogeneous carbon tax (panel b) exhibits significant variation, with rates for high-productivity households starting at $400 per ton, reflecting their lower marginal utility of consumption.

Both the heterogeneous and uniform policies achieve significant temperature reductions relative to business-as-usual (BAU). The heterogeneous carbon tax induces the strongest reduction in carbon accumulation, resulting in a long-run temperature reduction of 1.1 degrees relative to the BAU (Figure C.5).

Figure C.5: Carbon and temperature


(a) Carbon stock

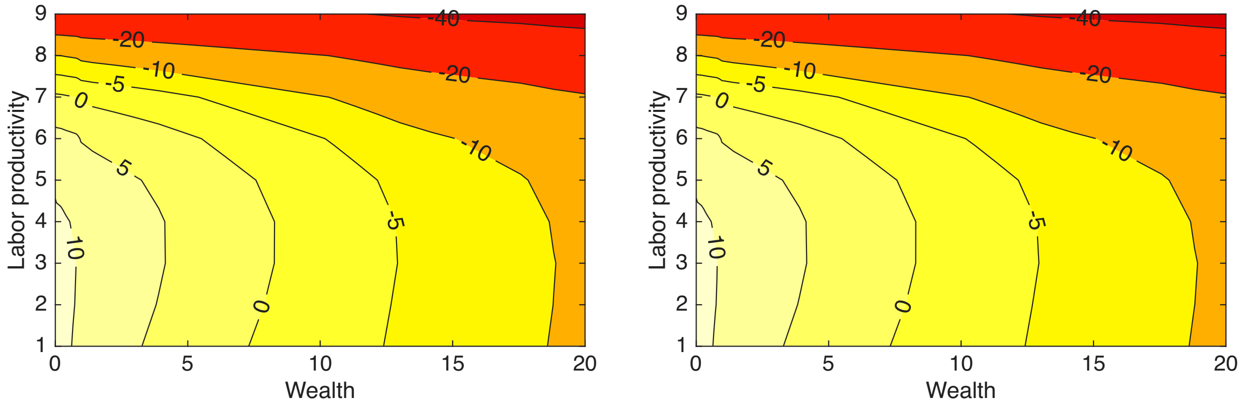
(b) Temperature increase

Both constrained-efficient policies lead to large welfare changes across the wealth distribution (Figure C.6). In either the uniform carbon tax (panel a) or the heterogeneous carbon tax (panel b) case, welfare gains are positive at the bottom of the income and wealth distribution and negative at the top of the income and wealth distribution, with very large magnitudes at the extremes.

Table C.3 summarizes the aggregate welfare and climate implications. A striking result is the magnitude of the welfare gains under the constrained-efficient policy compared to carbon taxation in the laissez-faire economy. While introducing only a carbon tax to the laissez-faire economy yields a modest average welfare gain of roughly 0.05 percent, implementing the CEA yields a gain of 2.33 percent.

Compared to the baseline, average welfare is several orders of magnitude greater under the CEA. This can be seen in Figure C.7 which decomposes the average welfare effects over time into economic and climate components. In contrast to our baseline (Figure 5 in Section 5.4) where the economic effects (red area) were exclusively negative, under the CEA they are positive and comprise the bulk of the welfare gains. Climate benefits (blue area) on the other

Figure C.6: Welfare (consumption equivalents, percent)


(a) Uniform

(b) Heterogeneous

Notes: Wealth is in units of per capita GDP. Labor productivity corresponds to the nine discrete states of the persistent idiosyncratic productivity process. Welfare gains are expressed in consumption equivalents (percent) relative to the Business-as-Usual (BAU) scenario.

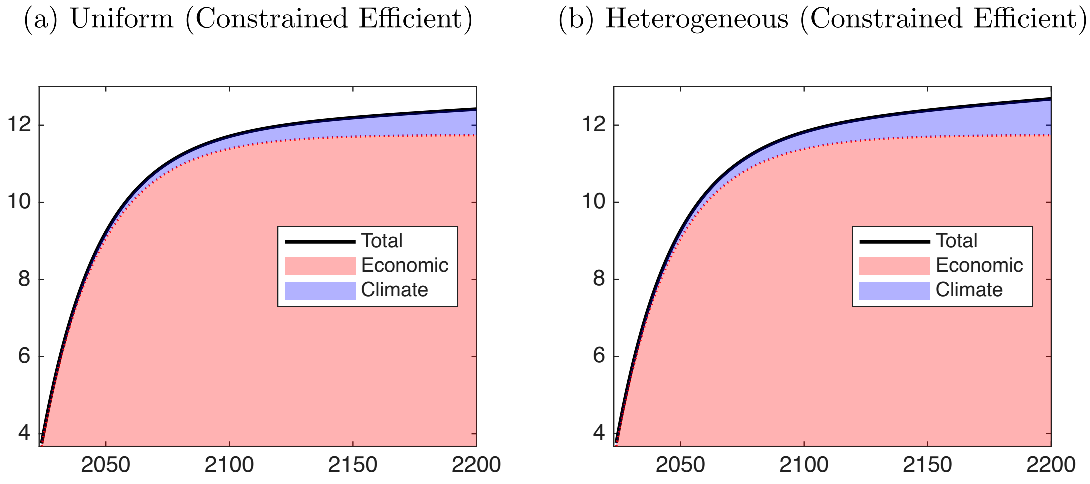
Table C.3: Welfare and Climate

Policy	Initial carbon tax ($/ton)	Long-run temp. reduction (degrees)	Average welfare gain (percent)	Support (percent)
Heterogeneous tax (CTO)	69.8*	1.3	0.05	99.73
Heterogeneous tax (CEA)	43.7*	1.1	2.33	70.95
Uniform tax (CTO)	32.7	0.8	0.04	100.00
Uniform tax (CEA)	26.4	0.8	2.33	70.93

Note: * denotes the average carbon tax rate, weighted by dirty consumption expenditures.

hand are relatively small, accumulating very gradually as the stock of atmospheric carbon diverges from the business-as-usual path.

Figure C.7: Average welfare over time



(a) Uniform (Constrained Efficient)

(b) Heterogeneous (Constrained Efficient)

Units: percent.

This large welfare gain is quantitatively consistent with the findings in Section 5.2 of Davila et al. (2012) regarding the "High Wealth Dispersion" economy. In both that model and ours, the consumption-poor derive most of their income from labor. Because markets are incomplete, the laissez-faire equilibrium features an underaccumulation of capital relative to the constrained optimum. The planner corrects this pecuniary externality by subsidizing savings—as shown previously in Figure C.2(a)—which deepens the capital stock, raises wages, and lowers interest rates. This shift in factor prices acts as a powerful insurance mechanism for the poor, who rely heavily on labor income. Consequently, the bulk of the 2.33 percent welfare gain in the CEA arises not from climate mitigation, but from the planner's ability to internalize the pecuniary externality and correct the capital underaccumulation inherent in the high-inequality market equilibrium. Notably, however, the optimal carbon tax schedule remains quantitatively similar whether these broader fiscal distortions are corrected or not. Motivated by this robustness, the analysis in the main text focuses on studying the optimal carbon tax while taking the existing U.S. labor and capital taxes as given.

D Mathematical Appendix

Proof of Lemma 1. The proof consists of showing that the conditions for the optimal allocation satisfy the conditions of a competitive equilibrium. It follows from a simple observation of optimality conditions (13) and (49) that both coincide when τ_t is replaced by the optimal tax as specified in (15) where σ_t is defined as in (9).

Second, substitute equilibrium prices $w_t = p_t = 1$ into (44). It is easy to check that the government's budget constraint holds. Finally, (44) holding for all i and the government constraint being satisfied imply that resource feasibility (4) holds. □

Proof of Proposition 1. The proof consists of showing that the conditions for the constrained optimal allocation—characterized by equations (9), (19), and (23), together with feasibility constraints and the carbon cycle—satisfy the conditions of a competitive equilibrium, characterized by equations (44)–(49) with taxes τ_t^i as in (24) and transfers that satisfy (18).

First, comparing (23) and (49), simple observation shows that both coincide when τ_t^i is replaced by the optimal tax as specified in (24), $\tau_t^i = \frac{\upsilon\sigma_t}{u_{ct}^i}$ where σ_t is defined as in (9).

Second, substitute (18) into individual budget constraints in (44) with $w_t = p_t = 1$ to obtain

$$c_{ct}^i + c_{dt}^i = \varepsilon^i \tag{71}$$

Thus, (44) holds at the optimal allocation as (71) coincides with the implementability condition in (19). It is easy to check that the government's budget constraint holds, plugging (18) into (45) and $p_t = 1$. Finally, by (71) holding for all i and the government constraint being satisfied, market clearing conditions hold. □

Proof of Proposition 2. The proof follows from demonstrating that the equations characterizing the uniform constrained-efficient planner's problem are equivalent to the equations characterizing the competitive equilibrium, characterized by equations (44)–(49) with taxes and transfers as in (28), together with market clearing conditions.

The first-order conditions for the constrained social planner's problem are:

$$(c_{dt}^i) : \mu_i u_{dt}^i - \upsilon\mu_i\sigma_t - \mu_i\lambda_t^i + \sum_{j\neq i}\eta_t^{ij}\left(c_{ct}^j + \bar{c}\right) - \sum_{j\neq i}\eta_t^{ji}\left(c_{ct}^j + \bar{c}\right) = 0 \tag{72}$$

$$(c_{ct}^i) : \mu_i u_{ct}^i - \mu_i\lambda_t^i - \sum_{j\neq i}\eta_t^{ij}c_{dt}^j + \sum_{j\neq i}\eta_t^{ji}c_{dt}^j = 0 \tag{73}$$

$$(S_{t+1}) : -\beta^t x'(S_{t+1}) + \sigma_t\beta^t - \sigma_{t+1}\beta^{t+1}(1-\delta) = 0 \tag{74}$$

$$(\beta^t\lambda_t^i) : c_{ct}^i + c_{dt}^i = \varepsilon_i \tag{75}$$

$$(\beta^t\eta_t^{ij}) : (c_{ct}^i + \bar{c})c_{dt}^j = (c_{ct}^j + \bar{c})c_{dt}^i \tag{76}$$

where $\beta^t\eta_t^{ij}$ is the Lagrange multiplier on the constraint on allocations, (27). Iterating forward from (74), we have:

$$\sigma_t = \sum_{j=1}^{\infty} [\beta(1-\delta)]^{j-1} x'(S_{t+j}). \tag{77}$$

Combine equations (72) and (73) to obtain:

$$\frac{u^i_{dt}}{u^i_{ct}} - 1 = \frac{1}{\mu_i u^i_{ct}} \left[\upsilon \mu_i \sigma_t - \sum_{j \neq i} \eta^{ij}_t (c^j_{ct} + \bar{c} + c^j_{dt}) + \sum_{j \neq i} \eta^{ji}_t (c^j_{ct} + \bar{c} + c^j_{dt}) \right] \tag{78}$$

Multiplying both sides of equation (78) by $\mu_i u^i_{ct}(c^i_{ct} + \bar{c} + c^i_{dt})$ and sum across all i, we obtain:

$$\begin{aligned} \sum_i \left(\frac{u^i_{dt}}{u^i_{ct}} - 1 \right) \mu_i u^i_{ct} \left(c^i_{ct} + \bar{c} + c^i_{dt} \right) =& \upsilon \sigma_t \sum_i \mu_i \left(c^i_{ct} + \bar{c} + c^i_{dt} \right) \\ & - \sum_i \left(c^i_{ct} + \bar{c} + c^i_{dt} \right) \sum_{j \neq i} \eta^{ij}_t (c^j_{ct} + \bar{c} + c^j_{dt}) \\ & + \sum_i \left(c^i_{ct} + \bar{c} + c^i_{dt} \right) \sum_{j \neq i} \eta^{ji}_t (c^j_{ct} + \bar{c} + c^j_{dt}) \\ =& \upsilon \sigma_t \sum_i \mu_i \left(c^i_{ct} + \bar{c} + c^i_{dt} \right) \end{aligned} \tag{79}$$

where the second equality can be shown by algebra manipulation.

Equation (25) implies that the marginal rate of substitution between dirty and clean consumption must be equal for all i. Thus we can simplify equation (79) to obtain

$$\frac{u^i_{dt}}{u^i_{ct}} = 1 + \tau_t \tag{80}$$

where

$$\tau_t = \frac{\upsilon \sigma_t}{\sum_i \frac{\mu_i c^i_t}{\sum_j \mu_j c^j_t} u^i_{ct}} \tag{81}$$

and $c^i_t \equiv c^i_{ct} + \bar{c} + c^i_{dt}$. Comparing this equation with the market intratemporal condition (49) leads to (28).

Second, substitute $T^i_t = \tau_t c^i_{dt}$ into individual budget constraints in (44) with $w_t = p_t = 1$ to obtain

$$c^i_{ct} + c^i_{dt} = \varepsilon^i, \tag{82}$$

which holds as it coincides with (19).

Finally, given that the budget constraint holds for all i and the government budget constraint holds, market clearing conditions are trivially satisfied. □

Proof of Corollary 1. The proof consists of showing that the competitive equilibrium conditions with taxes and transfers specified by (31) are satisfied by the constrained-efficient allocation.

First, evaluate the intratemporal consumption decision (30) at the taxes in (31) to get

$$\frac{u^i_{dt}}{u^i_{ct}} = \frac{1+\gamma\tau_t}{1-(1-\gamma)\frac{\tau_t}{1+\tau_t}}, \tag{83}$$

which simplifies to

$$\frac{u^i_{dt}}{u^i_{ct}} = 1+\tau_t. \tag{84}$$

which holds as it coincides with (80).

Second, to see that the budget constraint is satisfied, substitute the taxes and transfers in (31) into (29), to get

$$(1+\gamma\tau_t)c^i_{dt} + \left[1-(1-\gamma)\frac{\tau_t}{1+\tau_t}\right]c^i_{ct} = \varepsilon^i + \tau_{ct}\bar{c}, \tag{85}$$

which simplifies to

$$\begin{aligned} c^i_{dt} + c^i_{ct} - \varepsilon^i &= \frac{\tau_t}{1+\tau_t}\left\{(1-\gamma)(c^i_{ct}+\bar{c}) - \gamma c^i_{dt}(1+\tau_t)\right\} \\ &= 0. \end{aligned} \tag{86}$$

The last equality results from the preferences specified in (26), in which case, (30) implies

$$(1-\gamma)\left(c^i_{ct}+\bar{c}\right) = \gamma c^i_{dt}(1+\tau_t). \tag{87}$$

Market equilibrium conditions (46)-(48) hold as the government budget constraint is satisfied and (86) holds for all i. □

Proof of Proposition 3. The first part of the proof comes from comparing the intratemporal optimality condition on clean and dirty consumption in the quantitative model economy – which is the same as in the simple model, equation (49) – with the constrained optimal on (61). Both coincide if the carbon tax equals

$$\tau^i_t = \frac{\upsilon\sigma_t}{u^i_{ct}}$$

It follows that the constrained optimal carbon tax equals the one characterized in Proposition 1.

To show that income taxes take the form in the proposition, consider the equilibrium optimality conditions for labor and savings decisions:

$$\frac{v^i_{nt}}{u^i_{ct}} = (1 - T'^n_t) F_{Nt} \varepsilon^i_t \tag{88}$$

$$u^i_{ct} \geq \beta \mathbf{E}_t \left\{ u^i_{c,t+1} \left[(1 - \tau^k_{t+1}) F_{K,t+1} + 1 - \delta_k \right] \right\} \tag{89}$$

with the last equation holding with equality if the borrowing constraint is not binding. We consider a flat labor income tax, a capital income tax over total capital income (including depreciation), both of which may be individual-specific. We also assume that the borrowing constraints are not binding. These assumptions are without loss of generality to obtain simple tax formulas. Thus, the market optimality conditions can be written as

$$\frac{v^i_{nt}}{u^i_{ct}} = (1 - \tau^i_{nt}) F_{Nt} \varepsilon^i_t \tag{90}$$

$$u^i_{ct} = \beta \mathbf{E}_t \left\{ u^i_{c,t+1} \left[(1 - \tau^i_{k,t+1})(F_{K,t+1} + 1 - \delta_k) \right] \right\} \tag{91}$$

Comparing the constrained-optimal Euler equation (60) with the market optimality condition (91), it follows that both coincide when the capital income tax/subsidy equals

$$\tau^i_{k,t+1} = \Lambda^i_{k,t+1} \tag{92}$$

Similarly, a labor income tax/subsidy is required to implement the constrained-optimal allocation. The optimal tax rate comes from comparing the constrained optimality condition (59) with (90) and equals

$$\tau^i_{nt} = \Lambda^i_{nt} \tag{93}$$

Tax revenues are rebated lump-sum to each household:

$$\tau^i_{nt} F_{Nt} \varepsilon^i_t n^i_t + \tau^i_{kt} (F_{Kt} + 1 - \delta) k^i_t + \tau^i_{dt} c^i_{dt} = T^i_t \tag{94}$$

so that the budget constraint of the government holds for every period t.

Given that the budget constraint holds for all i and the government budget constraint holds, market clearing conditions are trivially satisfied. This completes the proof. □

Proof of Corollary 2. To prove that the uniform carbon tax takes the form in (28), combine equations (66) and (67) to obtain:

$$\frac{u^i_{dt}}{u^i_{ct}} = 1 + \frac{1}{\alpha_i u^i_{ct}} \left[\upsilon \mu_i \sigma_t - \sum_{j \neq i} \eta^{ij}_t (c^j_{ct} + \bar{c} + c^j_{dt}) + \sum_{j \neq i} \eta^{ji}_t (c^j_{ct} + \bar{c} + c^j_{dt}) \right] \tag{95}$$

where the social cost of carbon, σ_t, is given by (62). Uniform taxation implies that the marginal rate of substitution between dirty and clean consumption must be equal for all i. Thus,

$$\frac{u^i_{dt}}{u^i_{ct}} = 1 + \tau_t \tag{96}$$

Following the proof of Proposition 2, plug this into equation (95) and multiplying by $\mu_i u^i_{ct}(c^i_{ct} + \bar{c} + c^i_{dt})$ and sum across all i to obtain:

$$\begin{aligned} \sum_i \tau_t \mu_i u^i_{ct} \left(c^i_{ct} + \bar{c} + c^i_{dt}\right) =& \upsilon \sigma_t \sum_i \mu_i \left(c^i_{ct} + \bar{c} + c^i_{dt}\right) \\ &- \sum_i \left(c^i_{ct} + \bar{c} + c^i_{dt}\right) \sum_{j \neq i} \eta^{ij}_t (c^j_{ct} + \bar{c} + c^j_{dt}) \\ &+ \sum_i \left(c^i_{ct} + \bar{c} + c^i_{dt}\right) \sum_{j \neq i} \eta^{ji}_t (c^j_{ct} + \bar{c} + c^j_{dt}) \\ =& \upsilon \sigma_t \sum_i \mu_i \left(c^i_{ct} + \bar{c} + c^i_{dt}\right) \end{aligned} \tag{97}$$

where the last equality comes from algebra manipulation. Reorganizing terms, we get

$$\tau_t \equiv \frac{\upsilon \sigma_t}{\sum_i \frac{\mu_i c^i_t}{\sum_j \mu_j c^j_t} u^i_{ct}} \tag{98}$$

where $c^i_t \equiv c^i_{ct} + \bar{c} + c^i_{dt}$. Thus, the uniform carbon tax takes the form of equation (28). ☐

Proof of Corollary 3. We want to show that the uniform constrained-efficient allocation can be decentralized using the uniform instruments $\{\tau_{dt}, \tau_{ct}, T^{clim}_t\}$ alongside individual income taxes and rebates.

Consider the household budget constraint (33), modified to include the uniform clean subsidy τ_{ct} and two distinct lump-sum transfers: a uniform climate dividend T^{clim}_t and a household-specific income tax rebate T^i_t.

$$(1 + \tau_{dt}) p_t c^i_{dt} + (1 - \tau_{ct}) c^i_{ct} + k^i_{t+1} \leq w_t \varepsilon^i_t n^i_t (1 - \tau^i_{nt}) + R_t k^i_t (1 - \tau^i_{kt}) + T^{clim}_t + T^i_t$$

Let the uniform policy instruments be defined as in Corollary 1: $\tau_{dt} = \gamma \tau_t$, $\tau_{ct} = \frac{(1-\gamma)\tau_t}{1+\tau_t}$, and $T^{clim}_t = \tau_{ct} \bar{c}$.

First, regarding the intratemporal consumption choice, the household equates the marginal rate of substitution to the ratio of after-tax prices (with $p_t = 1$):

$$\frac{u^i_{dt}}{u^i_{ct}} = \frac{1 + \tau_{dt}}{1 - \tau_{ct}} = \frac{1 + \gamma \tau_t}{1 - \frac{(1-\gamma)\tau_t}{1+\tau_t}} = 1 + \tau_t$$

This ensures the allocation satisfies the uniform wedge condition required by the planner.

Second, consider the fiscal burden of the climate instruments. Given our assumption on preferences in (26) optimal consumption satisfies

$$(1-\gamma)(c^i_{ct}+\bar{c}) = \gamma c^i_{dt}(1+\tau_t). \tag{99}$$

The net climate tax payment by the household is:

$$\mathcal{T}^{clim}_{it} = \tau_{dt}c^i_{dt} - \tau_{ct}c^i_{ct} - T^{clim}_t$$

Substituting the definition of τ_{dt} and T^{clim}_t and rearranging:

$$\mathcal{T}^{clim}_{it} = \gamma\tau_t c^i_{dt} - \tau_{ct}(c^i_{ct}+\bar{c})$$

Using the definition of τ_{ct} and the optimal consumption relation in (99):

$$\mathcal{T}^{clim}_{it} = \gamma\tau_t c^i_{dt} - \left(\frac{(1-\gamma)\tau_t}{1+\tau_t}\right)\left(\frac{\gamma c^i_{dt}(1+\tau_t)}{1-\gamma}\right) = 0$$

Thus, the uniform climate policy is budget-neutral for every household.

Third, the intratemporal condition for labor and the intertemporal condition for savings given the tax policy are now given by:

$$\frac{v^i_{nt}}{u^i_{ct}} = \frac{1-\tau^i_{nt}}{1-\tau_{ct}}F_{Nt}\varepsilon^i_t \tag{100}$$

$$\frac{u^i_{ct}}{1-\tau_{ct}} = \beta\mathbf{E}_t\left\{\frac{u^i_{c,t+1}}{1-\tau_{c,t+1}}\left[(1-\tau^i_{k,t+1})(F_{K,t+1}+1-\delta_k)\right]\right\} \tag{101}$$

To implement the efficient levels of labor and capital, set the individual income taxes τ^i_{nt} and τ^i_{kt} so that (100) and (101) coincide with (59) and (60). The implied income taxes are then given by:

$$1-\tau^i_{nt} = (1-\tau^i_{ct})(1-\Lambda^i_{nt}) \tag{102}$$

$$1-\tau^i_{k,t+1} = \frac{1-\tau_{ct}}{1-\tau_{c,t+1}}(1-\Lambda^i_{k,t+1}) \tag{103}$$

To satisfy the planner's no-transfer constraint, set the individual income tax rebate equal to the revenue generated by these taxes:

$$T^i_t = \tau^i_{nt}w_t\varepsilon^i_t n^i_t + \tau^i_{kt}R_t k^i_t$$

Substituting the zero net climate burden and the definition of T_t^i into the budget constraint eliminates all tax and transfer terms, yielding:

$$c_{dt}^i + c_{ct}^i + k_{t+1}^i \leq w_t \varepsilon_t^i n_t^i + R_t k_t^i$$

This is identical to the planner's implementability constraint (51). Since the household faces the same effective relative prices and the same resource constraint as the planner, the household chooses the uniform constrained-efficient allocation. □

E Sensitivity Analysis

In this section, we check the sensitivity of our main results—that a uniform carbon tax rebated back to households leads to a Pareto improvement—to alternative values of select parameters and to different model assumptions. First, we report how our results change for different coefficients of relative risk aversion. This parameter controls the degree to which the dispersion in consumption changes the uniform constrained-optimal carbon tax. Rows 2 and 3 of Table E.4 show that lower and higher risk aversion—all other parameters equal—lead to a higher and lower carbon tax path, respectively. Both cases maintain full support for carbon taxation, and each delivers similar average welfare gains compared to the baseline.

Next, we turn to the consequence of reducing the discount factor—all other parameters equal to the baseline. In this case, the optimal tax is lower than under the baseline because households (and the planner) discount future damages to the climate more aggressively. While this policy leads to smaller average welfare gains than in the baseline, it is still favored by all households.

Table E.4: Sensitivity Analysis

Policy	Initial carbon tax (\$/ton)	Long-run temp. reduction (degrees)	Average welfare gain (percent)	Support (percent)
Baseline	48.1	1.1	0.05	100.0
Low risk aversion ($\kappa = 1.5$)	54.9	1.2	0.08	100.0
High risk aversion ($\kappa = 2.5$)	42.9	1.0	0.05	100.0
Low discount factor ($\beta = 0.96$)	35.7	1.0	0.03	100.0
Heterogeneous damages ($\zeta = 0.12$)	48.1	1.1	0.07	100.0
Heterogeneous damages ($\zeta = -0.05$)	48.1	1.1	0.07	100.0

Heterogeneous Climate Damages. Up to this point, we have assumed that climate damages are uniformly distributed over households. Here we relax this assumption by allowing climate damages to be type-dependent. Specifically, we suppose that the utility loss from carbon is

$$x_i(S) = \frac{\Psi_i}{2} S^2, \tag{104}$$

where

$$\Psi_i = \Psi \frac{\varepsilon_i^{-\zeta}}{\sum_j \mu_j \varepsilon_j^{-\zeta}}.$$

The parameter ζ controls the degree to which climate damages depend on productivity. If $\zeta < 0$, then for a given level of the carbon stock, the utility loss rises with income, while the loss is larger for the poor if $\zeta > 0$. Setting $\zeta = 0$ corresponds to the baseline case. This functional form has the useful property that changes in ζ do not alter the equilibrium or optimal policies but only change how welfare is distributed across households.

To discipline the value of ζ, we use survey data on willingness-to-pay (WTP) measures from Kotchen et al. (2013). In this survey, households are asked how much they would be willing to pay each year to reduce carbon emissions by 17 percent. The survey also collects a range of demographic information, including income, age, education, and household size. To convert these WTP measures into consumption equivalents, we take the following steps. First, using the CEX, we regress household log consumption on log income and other household characteristics such as age, education, and household size. Then, we use the regression coefficients to infer consumption expenditures for each household in the WTP survey. Finally, we calculate each household's consumption equivalent by dividing its WTP by its consumption.

We compute the semi-elasticity of consumption equivalent with respect to log income to be -0.01 and calibrate ζ so that the model matches this semi-elasticity. This requires $\zeta = 0.12$, meaning that a 50 percent reduction in income is associated with a 9 percent increase in climate damage. In the baseline case ($\zeta = 0$), the semi-elasticity is 0.005, meaning that higher income households gain more from climate mitigation, because they value consumption less. We also consider the case where $\zeta = -0.05$, under which the semi-elasticity is 0.01. In this case, the welfare gains are skewed even more to the rich as they suffer larger climate damages.

The results from these specifications are reported in the fifth and sixth rows of Table E.4. On net, the cases with heterogeneous climate damage lead to changes in average welfare similar to the case with homogeneous climate damage, and, importantly, the uniform carbon tax and rebate policies are Pareto improving in all cases. The distribution of welfare gains from both heterogeneous climate damage cases can be seen in Figure E.8. As can be expected, the welfare gains are decreasing in income when climate damages are larger for the poor (panel a) and are increasing with income when climate damages are larger for the rich (panel b).

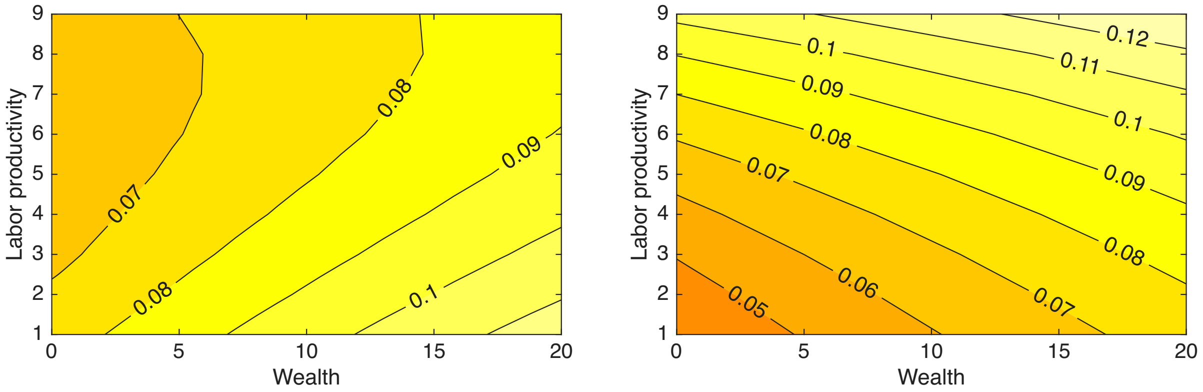


Figure E.8: Welfare with heterogeneous climate damages

(a) $\zeta = 0.12$

(b) $\zeta = -0.05$